Non-Linear Employment Effects of Tax Policy*

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Abstract

We study the non-linear propagation mechanism of tax policy in the context of a heterogeneous-agent equilibrium business cycle model with search frictions in the labor market and an extensive margin of employment adjustment. The model exhibits endogenous job destruction and endogenous hiring standards in the form of occasionally-binding zero-surplus constraints. We parametrize the model using U.S. data, including narratively-identified impulse response functions from proxy-SVARs. We find that the dynamic response of the employment rate to a temporary change in the flat-rate tax on labor income is highly non-linear, displaying sizable asymmetries and state-dependence. Notably, the response to a tax rate cut is at least twice as large in a recession than in an expansion.

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1 Introduction

This paper studies the *non-linear* propagation mechanism of discretionary tax policy in the context of a heterogeneous-agent equilibrium business cycle model featuring search frictions in the labor market, and an extensive margin of employment adjustment, that operates through endogenous job destruction and hiring. We ask three questions related to the effects of shocks to flat-rate labor taxes on the aggregate employment rate:

(i) Is the effect of a tax rate hike larger than the effect of a tax rate cut?

(ii) Does the *marginal* effect of a tax rate change depends on the size of the tax rate increase or decrease?

(iii) Is the effect of a tax rate cut larger in a recession than in an expansion?

Using a quantitative version of our model, we find that the answer to these questions is "yes." Overall, the *interaction* of search frictions and worker heterogeneity in skills produces significant non-linearities in the propagation mechanism of tax policy to the aggregate employment rate. Three main results stand out. First, we find that a cut in tax rates increases the employment rate by less than a tax rate hike reduces it, and that the larger the size of the shock, the larger the sign asymmetry between negative and positive tax shocks. Second, the marginal effect of a tax shock is decreasing in the size of the shock. Third, the effects of tax changes are state-dependent. Notably, the effect to a tax rate cut is at least twice as large in a recession than in an expansion.

Our results have important implications for a number of fiscal policy issues, including, but not limited to, countercyclical tax policy: to the extent that the marginal effect of a tax rate cut is decreasing in its size, tax policy is less effective as a countercyclical policy instrument than the estimates based on *linear* structural vector autoregressions (SVARs) would imply.

Our theoretical analysis builds on two premises. First, a typical U.S. recession is "L-shaped," as opposed to "V-shaped," featuring a sharp drop in the aggregate employment rate, followed by a recovery phase in which the employment rate slowly reverts back to its pre-recession level. These patterns clearly point to the presence of significant *non-linearities* in aggregate dynamics (McKay and Reis, 2008; McQueen and Thorley, 1993; Morley and Piger, 2012; Neftçi, 1984; Sichel, 1993). The Great Recession of 2008-09 and the contraction led by the spread of the new coronavirus (COVID-19) in early 2020, are the two most recent episodes of this phenomenon. We view endogenous job destruction as a key feature of the propagation mechanism of macroeconomic shocks, including tax shocks.

Second, congestion effects due to random search in the labor market, exacerbated by a disproportionate inflow of workers in the unemployment pool at the onset of recessions, can induce *convexity* in hiring costs, and thereby a high degree of curvature in the cost of producing output. This curvature leads to interesting non-linearities, implying that the responsiveness of the economy to discretionary tax policy may considerably vary over the business cycle, and its efficacy depend on the magnitude of the tax rate change itself.

Prominent features of our model are (i) search frictions, that give rise to equilibrium unemployment, as in the Diamond-Mortensen-Pissarides (DMP) framework (Diamond, 1982; Mortensen, 1982; Pissarides, 1985), and (ii) worker heterogeneity in productivity or skills, which delivers an extensive margin of employment adjustment.[1] Tax rates impinge on equilibrium allocations via two channels. First, they alter the relative return of market to non-market activity, as in the standard model of labor supply with home production. The second channel operates through the effective bargaining power of the worker. That is, the higher the tax rate on labor income, the lower the share of the surplus generated by a match accruing to the worker.

The model features endogenous separation and hiring in the form of occasionally-binding zero-surplus constraints: given a value for the tax rate and a level of aggregate productivity, a zero-surplus constraint defines a cutoff on worker productivity, such that existing matches with workers whose productivity is below the cutoff are endogenously destroyed. Analogously, meetings between employers posting vacancies and workers whose productivity is below the cutoff are not converted into jobs. In this sense, the model exhibits endogenous hiring standards in that who gets hired depends on tax rates as well as aggregate productivity.

Given these features, the model embeds two mechanisms whose interaction can yield highly non-linear effects of tax policy. First, absent search frictions, the model collapses to a *frictionless* setting with indivisible labor and heterogeneous workers. A reservation wage rule determines the extent to which available labor services are fully utilized or

[1] Recent empirical work points to the importance of worker heterogeneity for aggregate labor-market dynamics (Ahn and Hamilton, 2019; Barnichon and Figura, 2015).

left idle. More specifically, if the prevailing wage is above the reservation wage, the individual is employed and producing output, otherwise he/she is unemployed. This mechanism generates V-shaped responses to tax rate hikes: sharp contractions followed by quick recoveries. Second, search frictions, the extent of which varies over the business cycle, impede the instantaneous creation of jobs, leading to gradual recoveries. This mechanism induces L-shaped type responses to tax rate changes.

The dynamic response of the aggregate employment rate to changes in tax rates is driven by two forces. First, the extent of search frictions, as measured by the market tightness ratio, varies in response to tax shocks. Specifically, the tightness ratio, and so the probability that an unemployed worker bumps into a job vacancy, falls in response to a tax rate hike. Further, because the job-meeting probability is concave in the tightness ratio, it drops more in response to a tax rate hike than it raises in response to an equally-sized tax rate cut. Note also that the stimulative, positive effect of a tax rate cut on employment is dampened by the presence of search frictions. In the absence of frictions, the employment rate jumps to its new steady-state level insofar as the tax rate cut is large enough to make workers willing to supply labor.

Second, a tax rate hike makes the zero-surplus constraint bind, implying an immediate adjustment in employment through endogenous job destruction. Further, the larger the tax rate increase, the larger the fraction of workers hitting the zero-surplus constraint, the more important is the active margin of job destruction. This mechanism is a key driver of state-dependence in tax policy. Specifically, during recessions, a larger fraction of low productivity workers is at the margin, so that even a small tax rate cut can induce large responses in the aggregate employment rate.

To quantify these mechanisms, we parametrize the model using U.S. data, including impulse response functions (IRFs) from proxy-SVARs. Along with other data moments, the model reproduces the "peak" response of the U.S. employment rate (one minus the unemployment rate) to a narratively-identified shock to the average marginal tax rate (AMTR). In the model, consistently with the IRF from proxy-SVARs, a one percentage point (ppt), temporary reduction in the tax rate leads to about a 0.65 ppt increase in the employment rate.[2]

[2]Following the vast literature on SVARs, here we confront the model with the evidence on unanticipated AMTR shocks. See Mertens and Ravn (2012) for empirical evidence on the effects of anticipated tax policy shocks in the United States.

We use the quantitative model to gauge the extent of non-linearities in tax policy: non-linearities are pervasive, ranging from significant sign and size asymmetries to state-dependence. It is worth emphasizing that such non-linear responses arise from shocks whose magnitudes are comparable to those observed historically in the United States.

First, sign asymmetry is sizable. In response to a 1.5 ppt tax rate cut, the employment rate raises by nearly 0.45 ppt, whereas in response to an equally-sized tax rate hike, the employment rate falls by 0.8 ppt. Such asymmetry increases with the size of the shock: in response to a 3 ppt tax rate change, the drop in the employment rate is nearly three times as large as the the peak response to a tax rate cut.

A tax rate hike induces endogenous job destruction on impact, followed by a period of depressed vacancy posting. So, the pool of unemployment raises and at the same time, the probability of bumping into a job vacancy and finding a job falls. In this sense, tax rate hikes exacerbate the extent of search frictions. In response to a tax rate cut, instead, the adjustment comes about an increase in vacancy posting leading to a sluggish and hump-shaped increase in the employment rate.

Second, size asymmetry is also quantitatively important. For example, at the peak, the employment rate increase to a 3 ppt tax rate cut is approximately one-third of two times the increase in the employment rate to a 1.5 ppt tax rate cut. In this sense, the marginal effect of a tax rate cut decreases sharply with the size of the tax cut. While size asymmetry is present for tax rate increases, too, it is much less pronounced relative to tax rate cuts.

Third, the stimulative effect on the aggregate employment rate of a tax rate cut is at least twice as large in a recession than in an expansion. In the model, during expansions, unemployment is of the frictional type. This implies that tax rate changes work through changes in the extent of frictions, as measured by the speed at which an unemployed worker bumps into a job opportunity. A tax rate cut causes an increase in vacancy posting that in turn leads to a higher probability of finding a job. However, during an expansion, vacancy posting is already high, the labor market is tight, so that the increase in vacancy posting induced by the tax cut is small.

During recessions, instead, tax rate changes operate through changes in the extent of frictions, as well as changes in the fraction of zero-surplus workers. A tax rate cut that is large enough can then make a large fraction of unemployed workers viable for hiring. In addition, vacancy posting, too, is more responsive to tax rate cuts during recessions than expansions. The interaction of these two mechanisms also implies that the extent of state-dependence raises with the depth of the recession. Notably, the deeper the recession, the more effective tax rate reductions are in stimulating aggregate employment.

The rest of the paper is organized as follows. In Section 2, we discuss the related literature. In Sections 3 and 4, we present the model and illustrate its main qualitative properties. In Sections 5 and 6, we parametrize the model and study some of its basic quantitative properties. In Section 7, we show non-linear IRFs to gauge the extent of non-linearities in the propagation mechanism of tax shocks. Finally, Section 8 concludes. Appendix A contains details on data sources, variables' construction, and estimation.

2 Related Literature

This paper contributes to the important and growing literature studying non-linearities in the propagation mechanism of fiscal policy. Empirically, a strand of this literature aims at identifying the causal effects of shocks to government spending and tax rates on macro variables, and the extent to which the magnitude of these effects depends on the sign and size of the fiscal policy shock, as well as on the state of the business cycle. Given the limited number of time-series observations on aggregate macro variables, and the high frequency of countercyclical fiscal policy changes, it is well-known that the identification of the effects of fiscal policy remains a challenge in linear econometric models, let alone in the context of nonlinear models. Perhaps not surprisingly, then, there is no consensus about the extent of asymmetry and state-dependence in the transmission of fiscal policy.

Early empirical studies find that government spending multipliers are significantly larger in recessions than in expansions (see, e.g., Auerbach and Gorodnichenko, 2012, 2013; Bachmann and Sims, 2012; Fazzari, Morley and Panovska, 2015).[3] Using a historical dataset of U.S. government spending, Ramey and Zubairy (2018) find that the spending multiplier is acyclical, thus calling into question the early empirical findings. Recently, Barnichon, Debortoli and Matthes (2019) find that the sign of the change in government spending matters for the size of the multiplier. Notably, a reduction in government spending has contractionary effects on economic activity with a multiplier above one, and largest during recessions, whereas an increase in government spending is associated

[3] A "multiplier" is defined as the change in output caused by a $1 change in government purchases.

with a multiplier below one, which is virtually the same in recessions and in expansions. These findings establish the presence of both asymmetric and state-dependent effects of contractionary fiscal shocks.

On the theory side, there is relatively little work. Michaillat (2014) proposes a theory of a countercyclical government spending multiplier, using the "rationing unemployment" framework developed in Michaillat (2012). In Michaillat's model, job rationing emerges in equilibrium because firms face decreasing returns to labor and wages are rigid: after a negative technology shock, the marginal product of labor falls, but rigid wages adjust downward only partially. Thus, if the adverse shock is sufficiently large, the marginal product falls below the wage so that it is unprofitable for firms to hire workers regardless of the cost of posting a job vacancy.

Pizzinelli, Theodoridis and Zanetti (2020) show that a DMP model with endogenous job separation and on-the-job search replicates the empirical findings from a Threshold Vector Autoregression (TVAR) model that the unemployment rate, job-separation rate, and the job-finding rate exhibit a larger response to productivity shocks during periods when aggregate productivity is low. Ghassibe and Zanetti (2020) study the implications of search frictions in the product market for state-dependence in fiscal policy. They find that the magnitude and the variation of the fiscal multiplier over the business cycle depends on the source of economic fluctuations, that is on whether the economy is hit by demand or supply shocks. Finally, Fernández-Villaverde et al. (2020) develop a DMP model with search complementarities in the inter-firm matching process in which an equilibrium with high output and low unemployment coexists with another equilibrium with low output and high unemployment. In their model, fiscal policy is considerably more potent in the bad equilibrium characterized by low output and high unemployment.

We contribute to the theoretical strand of the literature in two ways. First, we propose a new mechanism for the non-linear effects of fiscal policy, based on worker heterogeneity and search frictions. More specifically, we stress the role of endogenous job destruction, as well as endogenous hiring standards, in the form of occasionally-binding zero-surplus constraints. The resulting extensive margin of employment adjustment interacts with search frictions, magnifying the extent of non-linearities. To the best of our knowledge, the importance of an active extensive margin subject to frictions has not been underscored by the literature.

Second, we analyze asymmetry and state-dependence in a unified framework, that encompasses dynamic responses to tax rate changes of different shapes depending on the relative importance of search frictions vis-à-vis the composition of the unemployment pool. Importantly, we discipline our quantitative model with narratively-identified IRFs, so that the model reproduces the peak response of the employment rate to an identified tax rate shock.

3 Model

In this section, we present the heterogeneous-agent equilibrium business cycle model that we later use to study the non-linear propagation mechanism of tax shocks. The model retains two main ingredients of the framework developed by Ferraro (2018): (i) worker heterogeneity in market productivity; (ii) search frictions in the labor market. We begin by presenting the economic environment: preferences, budget constraints, technology, market structure, and the stochastic process for tax shocks. We then set up the value functions, discuss wage determination, and finally turn to the equilibrium of the model.

3.1 Environment

Time is discrete and continues forever, indexed by $t = 0, 1, \ldots, \infty$. The model economy is populated by a continuum of measure one of workers, as well as by a continuum of employers or firms whose mass is determined in free-entry equilibrium. Workers and employers are infinitely-lived, risk-neutral, and discount future values at the same rate $0 < \beta < 1$. Workers are endowed with an indivisible unit of time per period, that is supplied as labor or used for job search. We adopt a two-state representation of the labor market, such that workers are either employed and producing output, or unemployed and searching for a job. Employers are either matched with a worker, and so producing output, or posting job vacancies to hire unemployed workers.

Heterogeneity Workers are heterogeneous in terms of their market productivity, which for lack of a better term we refer to as "skills." There are $M \geq 2$ skill types, indexed by $x \in X = \{x_1, x_2, \ldots, x_M\}$, where $x_1 < x_2 < \ldots < x_M$. We view the skill type x as an innate characteristic of the worker, that is known by employers with no uncertainty. The mass of workers of type x is denoted by $f(x)$, and the labor force is normalized to one, i.e. $\sum_{x \in X} f(x) = 1$. Identical employers have access to the same production technology, with labor services as the only input.

Preferences and budget constraints Workers' preferences over consumption $c_t \geq 0$ and time spent at work $n_t \in \{0, 1\}$ are described by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (c_t - B n_t), \tag{1}$$

where $\mathbb{E}_0$ is the expectation operator conditional on information available at $t = 0$ and $B \geq 0$ is the utility cost of working. Workers' budget constraint is $c_t \leq (1 - \tau_t) w_t n_t + (1 - n_t) \Gamma_t(w_t)$, where $(1 - \tau_t) w_t$ is the after-tax wage, $0 \leq \tau_t < 1$ is a flat-rate tax on labor income, and $\Gamma(w_t) \geq 0$ denotes government transfers to unemployed workers, such as unemployment insurance (UI) benefits.

Technology Production requires a match between one employer and one worker. When a match (or, equivalently, a "job") is created, output y is produced with a linear production function, $y = Ax$, where A denotes aggregate productivity, that we later use to parametrize the state of the business cycle, and x is the level of skills as defined above.

To capture the presence of search frictions, we assume that the number of meetings between unemployed workers and job vacancies is determined by a constant-returns-to-scale (CRS) meeting technology. Specifically, let $\theta \equiv v/u$ denote the ratio of job vacancies to unemployed workers, then an unemployed meets a job vacancy with probability $p(\theta) : \mathbb{R}_+ \to [0, 1]$. We further assume that $p(\theta)$ is strictly increasing and twice continuously differentiable with $p(0) = 0$ and $p(\infty) = 1$. Conversely, the probability that a vacancy meets an unemployed worker is $q(\theta) = p(\theta)/\theta$ with $q(0) = 1$ and $q(\infty) = 0$.

Market structure In the spirit of the "directed search" framework of Moen (1997), we consider a market structure featuring segmentation by skill types, so that the aggregate labor market consists of a collection of M submarkets indexed by x. Specifically, workers are assigned to different submarkets, and employers can post vacancies in one submarket at the time, directing their search by choosing the submarket with the highest expected payoff. In each submarket, then, the rate at which unemployed workers meet employers depends on the tightness ratio $\theta(x) \equiv v(x)/u(x)$, where $v(x)$ and $u(x)$ are job vacancies and unemployed workers in submarket x.

Beyond its theoretical appeal, this formulation of the market structure also captures a salient feature of the recruiting process. To streamline the application process, job vacancy postings typically include job qualifications as well as job requirements, specifying the skills, experience, and other attributes that the employer is seeking to find in the applicant who is hired for the job. Hence, in the model, as to a large extent in actual labor markets, the search process is in fact directed through vacancy posting.

Exogenous stochastic process for tax rates The flat-rate tax on labor income follows an AR(1) process in logs:

$$\log(\tau_{t+1}) = (1-\rho)\log(\bar{\tau}) + \rho\log(\tau_t) + \sigma\epsilon_{t+1}, \tag{2}$$

where $\bar{\tau}$ is the unconditional mean of the tax rate, and the parameters $0 < \rho < 1$ and $\sigma > 0$ govern the persistence of the shock and the volatility of the innovations $\epsilon_t \overset{iid}{\sim} N(0,1)$, respectively.

To be sure, in the United States, labor tax rates have changed over time for a number of reasons, including, but not limited to, for the response of the government to current business cycle conditions (see Romer and Romer, 2010, for a narrative account of postwar U.S. tax policy). Here, instead, we model the stochastic process for tax rates to capture the "exogenous" component of tax policy, akin to the structural shock approach in SVARs. Specifically, an innovation to the tax rate meets three requirements: (i) it is unpredictable given current and past information; (ii) it is uncorrelated with other potential structural shocks hitting the economy; (iii) it is unanticipated, namely, it is not news about future changes in tax rates.

Timeline Within a period, events unfold as follows. At the beginning of the period, the aggregate shock is realized. After this realization, the period has two stages. First, job separation, job creation, and search decisions are made simultaneously. Second, at the production stage, output is produced and wages are paid.

In the setting here, there is a separation between the time at which a meeting occurs and the formation of a match. During the period, job search and job vacancies determine meetings between workers and employers. At the beginning of next period, bilateral

Nash-bargaining takes place and if profitable a meeting is converted into a job. This formulation of the hiring process is consistent with the "non-sequential" search paradigm (see Van Ours and Ridder, 1992; Abbring and Van Ours, 1994; Ommeren and Russo, 2014; Davis and de la Parra, 2017).[4]

3.2 Value Functions

We formulate the problem of workers and employers in recursive form. Notably, we write the Bellman equations at the production stage, when the current decisions of continuing, destroying, or creating a match have already been made. All the information needed for optimal decision making is the realization of the tax rate τ, which is the state variable of the dynamic program.

Henceforth, we omit time subscripts and use primes to denote next period variables. Also, abusing notation slightly, we denote the probability that an unemployed worker meets a job vacancy by $p(x) \equiv p\left(\theta(\tau; x)\right)$, and the probability that a job vacancy meets an unemployed worker by $q(x) \equiv q\left(\theta(\tau; x)\right)$.

Employer Bellman equations At the production stage, the value of a job in submarket x satisfies the Bellman equation

$$J(\tau; x) = Ax - w(x) + \beta \mathbb{E}\left[\max_{d'(x)} \left\{ \left(1 - d'(x)\right) J(\tau', x) + d'(x) \max_{\tilde{x}} V(\tau'; \tilde{x}) \right\}\right], \qquad (3)$$

where Ax is the output of a match, $w(x)$ is the wage rate, whose determination we discuss below, and $\tilde{x}$ indicates the employer's choice about where to post a job vacancy.

The employer's decision to destroy the match is subsumed in the indicator function

$$d(x) = \begin{cases} \delta & \text{if} \quad J(\tau; x) > \max_{\tilde{x}} V(\tau; \tilde{x}) \\ 1 & \text{if} \quad J(\tau; x) \leq \max_{\tilde{x}} V(\tau; \tilde{x}), \end{cases} \qquad (4)$$

where $0 < \delta < 1$ is an exogenous rate of job destruction. Note that the max operator in (4)

[4] See Stigler (1961), Gal, Landsberger and Levykson (1981), Morgan (1983), and Morgan and Manning (1985) for early theoretical work on non-sequential search and Gautier (2002), Wolthoff (2014), and Albrecht, Gautier and Vroman (2006) for recent work on the efficiency properties of environments with non-sequential search.

captures the employer's decision to post a job vacancy in the submarket with the highest expected value. In this sense, search is directed.

At the search stage, the value of a job vacancy posted in submarket x satisfies the Bellman equation

$$V(\tau;x) = -k + \beta \mathbb{E}\Big[q(x) \max_{h'(x)} \Big\{h'(x)J(\tau';x) + \big(1-h'(x)\big) \max_{\tilde{x}} V(\tau';\tilde{x})\Big\} \\ + (1-q(x)) \max_{\tilde{x}} V(\tau';\tilde{x})\Big], \tag{5}$$

where $k \geq 0$ is the unit cost of posting and maintaining a job vacancy, and the employer's job creation decision is subsumed in the indicator function

$$h(x) = \begin{cases} 1 & \text{if} \quad J(\tau;x) > \max_{\tilde{x}} V(\tau;\tilde{x}) \\ 0 & \text{if} \quad J(\tau;x) \leq \max_{\tilde{x}} V(\tau;\tilde{x}). \end{cases} \tag{6}$$

Worker Bellman equations At the production stage, the value of being employed in submarket x satisfies the Bellman equation

$$W(\tau;x) = (1-\tau)w(x) - B + \beta\mathbb{E}\left[\max_{s'(x)}\left\{\big(1-s'(x)\big)W(\tau';x) + s'(x)U(\tau';x)\right\}\right], \tag{7}$$

where the worker's instantaneous return from working $(1-\tau)w(x) - B$ is the after-tax wage net of the disutility of work.

The worker's job separation decision is subsumed in the indicator function

$$s(x) = \begin{cases} \delta & \text{if} \quad W(\tau;x) > U(\tau;x) \\ 1 & \text{if} \quad W(\tau;x) \leq U(\tau;x). \end{cases} \tag{8}$$

At the search stage, the value of being unemployed in submarket x satisfies the Bellman equation

$$U(\tau;x) = \Gamma(w) + \beta\mathbb{E}\Big[p(x) \max_{a'(x)} \Big\{a'(x)W(\tau';x) + \big(1-a'(x)\big)U(\tau';x)\Big\} \\ + (1-p(x))\,U(\tau';x)\Big], \tag{9}$$

where $\Gamma(w)$ is government transfers, and the worker's job acceptance decision is subsumed in the indicator function

$$a(x) = \begin{cases} 1 & \text{if} \quad W(\tau; x) > U(\tau; x) \\ 0 & \text{if} \quad W(\tau; x) \leq U(\tau; x). \end{cases} \tag{10}$$

3.3 Equilibrium

As standard in the literature, we consider a free-entry equilibrium in which the value of posting a job vacancy equals zero in all submarkets at all times. This equilibrium concept, coupled with a directed search process, greatly simplifies the computation of the model as the Bellman equations, and thereby the total match surplus, depend solely on the tax shock τ, which captures all the relevant information for optimal decisions. As a result, individual agents' decision rules and tightness ratios do not depend on the distribution of workers over wages and unemployment across submarkets. The literature refers to an equilibrium with such properties as a Block Recursive Equilibrium (BRE) (see Shi, 2009; Menzio and Shi, 2010*a*,*b*, 2011).[5]

In addition, as in the standard DMP model, the equilibrium within each submarket is block-recursive, too. That is, we can solve for the equilibrium tightness ratio and decision rules independently of the stocks of employment or unemployment. Given these decision rules and tightness ratio, we then compute the evolution of the stocks of employment by calculating the flows of hires and separations.

To summarize, the equilibrium in each submarket is characterized by three equations: (i) a Bellman equation for the total match surplus generated by the match; (ii) a free-entry condition for vacancy posting; (iii) a wage equation that implicitly determines the split of the surplus between workers and employers. We solve for the equilibrium match surplus and tightness in each submarket separately, and then aggregate across submarkets.

[5] A BRE gives a considerable advantage over an equilibrium with random search. To see this, consider an alternative market structure in which markets are not segmented so that individuals with different levels of skills search in the same market. In such an environment, under Nash bargaining, vacancy-posting firms would need to keep track of the distribution of unemployed workers over skills. In a free-entry equilibrium, this implies that job-finding probabilities become a function of the distribution of skills, an infinitely dimensional object that cannot be characterized analytically.

3.3.1 Wage Setting

The wage is determined via bilateral Nash bargaining. When a worker and an employer meet, they bargain over how to split the surplus generated by the match. As standard in the literature, we assume that bargaining resumes every period so that workers in old and new matches receive the same wage. Specifically, in each submarket, the wage is determined to maximize the weighted product of the net match surplus accruing to the worker, $W(\tau; x) - U(\tau; x)$, and the net surplus accruing to the firm, $J(\tau; x)$, i.e.,

$$w(x) = \arg\max[W(\tau; x) - U(\tau; x)]^{\eta} J(\tau; x)^{1-\eta}, \tag{11}$$

where $0 \leq \eta \leq 1$ is a parameter reflecting the worker "bargaining power." In the case in which the bargaining set is empty, we set $w(x) = 0$.

The solution to the Nash-bargaining problem specified in (11) satisfies a modified sharing rule that takes in account the presence of the flat-rate tax on labor income:

$$\eta(1 - \tau) J(\tau; x) = (1 - \eta)[W(\tau; x) - U(\tau; x)]. \tag{12}$$

This equation specifies that shifts in the labor tax rate act like changes in the effective bargaining power of the worker. Specifically, the higher the tax rate, the smaller is the share of the total match surplus that goes to the worker.

To see this clearly, let $S(\tau; x) \equiv [W(\tau; x) - U(\tau; x)]/(1 - \tau) + J(\tau; x)$ denote the total match surplus generated by a match. Using the sharing rule (12), we obtain (after some algebra) that the value of a job and the net value of working are proportional to the total match surplus:

$$J(\tau; x) = (1 - \eta) S(\tau; x), \tag{13}$$

$$W(\tau; x) - U(\tau; x) = \eta(1 - \tau) S(\tau; x). \tag{14}$$

Note that equations (13) and (14) imply "bilateral efficiency." That is, employers and workers always agree on the decision to destroy an existing match, or to create a new one, implying that $d(x) = s(x)$ and $h(x) = a(x)$ for all possible realizations of the tax shock. In this sense, changes in tax rates do not induce inefficient separations. Yet, tax shocks will affect equilibrium outcomes by changing the total surplus generated by the match, as well as the split of this surplus between workers and employers.

3.3.2 Three Equations that Determine the Equilibrium

We now turn to describe the three equations that determine the equilibrium of the model.

Total match surplus At the production stage, the total match surplus generated by a match in submarket x satisfies the Bellman equation

$$S(\tau;x) = Ax - \frac{B+\Gamma(w(x))}{1-\tau} + \frac{\eta\beta}{1-\tau}\left[1-\delta-p(\theta(x))\right]\mathbb{E}\left[(1-\tau')\max\left\{S(\tau';x),0\right\}\right] + \beta(1-\delta)(1-\eta)\mathbb{E}\left[\max\left\{S(\tau';x),0\right\}\right]. \tag{15}$$

First, worker productivity Ax net of the *tax-adjusted* opportunity cost of employment, $B+\Gamma(w(x))/(1-\tau)$, (henceforth, "net productivity"), enters total match surplus as the instantaneous return to the match, such that shocks to τ act like exogenous shifts in the relative return to non-market work. Everything else equal, the higher the tax rate, the higher is the effective opportunity cost of employment, or the lower the relative return to market work.

Second, a shock to the current tax rate τ changes the conditional expectation of the tax rate next period τ'. Such a change in expectations matters for current decisions insofar as a higher (lower) tax rate next period (i) reduces (rises) expected match surplus, and (ii) lowers (rises) the expected share of the total surplus accruing to the worker.

This latter effect interacts with the level of slack in the labor market. The value of the market tightness ratio $\theta(x)$ affects, through the job-meeting probability $p\left(\theta(x)\right)$, the rate at which the individual worker and employer discount expected match surplus. This effect captures the option value of job search that the worker forgoes by accepting a job offer. The existence of this option value reduces total match surplus to the extent that the worker has some bargaining power, i.e. $\eta > 0$. Specifically, the lower the tightness ratio, the more important is the expectation of future tax rates for current decisions to destroy or create a match.

Free-entry condition Employers post job vacancies up to the point where the expected cost equals the expected benefit of opening and maintaining a job vacancy:

$$k \geq \beta(1-\eta)q(\theta(x))\mathbb{E}\left[\max\left\{S(\tau';x),0\right\}\right], \tag{16}$$

with equality in the case of an interior solution for job vacancies. This free-entry condition determines the tightness ratio in submarket x, independently of the other submarkets. In this setting, the distribution of unemployed workers over skills across submarkets plays no role in the determination of job vacancies. (At the aggregate level, the skill distribution of unemployment remains a key determinant of total hires.) Persistent shocks to tax rates affect tightness, and so the degree of slack in each submarket, through changes in the expected surplus, on the right-hand side of the free-entry condition (16). The higher the tax rate next period, the lower the expected surplus, which mandates a lower tightness ratio in the current period.[6]

Nash-bargained wages As in the standard DMP model, if government transfers do not depend on wages, i.e., $\Gamma(w(x)) = 0$, we can compute the equilibrium of the model by solving the Bellman equation for the total match surplus (15) and the free-entry condition (16). If instead $\Gamma(w(x)) > 0$, as in our case, then a wage equation has to be included and jointly solved with the surplus equation and the free-entry condition.

Using the sharing rule (13) and the equation for the value of a job (3), we obtain (after some algebra) a forward-looking wage equation:

$$w(x) = Ax - (1-\eta)S(\tau;x) + \beta(1-\eta)(1-\delta)\mathbb{E}\left[\max\left\{S(\tau';x),0\right\}\right]. \tag{17}$$

Next, using the free-entry condition (16), and the Bellman equation for the total match surplus (15), equation (17) becomes

$$w(x) = Ax + \frac{(1-\delta)k}{q(\theta(x))} - (1-\eta)S(\tau;x). \tag{18}$$

[6] Note that as long as expected surplus on the right-hand side of the free-entry condition is positive, the equilibrium features an interior solution for vacancy posting.

3.3.3 Individual Decision Rules and Aggregation

In each submarket, the joint employer-worker decision of creating a job, or destroying an existing one satisfies a cutoff rule. The zero-surplus constraint $S(\bar{\tau}; x) = 0$ yields cutoffs on tax rates $\bar{\tau}(x)$ that vary by skills, such that if $\tau \geq \bar{\tau}(x)$ a match with a worker of skill level x is destroyed, or, similarly, a meeting between a vacancy and an unemployed of skill level x is not converted into a job. Notably, the higher the skill level, the higher the cutoff on the tax rate that makes workers and employers indifferent between continuing or destroying an existing employment relationship, i.e. $\bar{\tau}(x_1) < \ldots < \bar{\tau}(x_M)$.

Individual decision rules More formally, in each submarket, endogenous job creation satisfies the decision rule

$$h(\tau; x) = a(\tau; x) = \begin{cases} 1 & \text{if} \quad S(\tau; x) > 0 \\ 0 & \text{if} \quad S(\tau; x) = 0. \end{cases} \tag{19}$$

Equivalently, endogenous job destruction satisfies the decision rule

$$d(\tau; x) = s(\tau; x) = \begin{cases} \delta & \text{if} \quad S(\tau; x) > 0 \\ 1 & \text{if} \quad S(\tau; x) = 0. \end{cases} \tag{20}$$

Aggregation In each submarket, the employment rate is

$$e'(x) = e(x) + h'(x)p\left(\theta(x)\right)u(x) - d'(x)e(x). \tag{21}$$

Summing across submarkets, the aggregate employment rate evolves over time based on the total flows of hires, $\Phi' U$, and separations, $\Delta' E$, such that

$$E' = E + \Phi' U - \Delta' E, \tag{22}$$

where $E = \sum_x f(x)e(x)$, and the aggregate job-finding rate (Φ) and job-separation rate

(Δ) next period are calculated as

$$\Phi' = (1/U) \sum_x h'(x) p(\theta(x)) h(x) u(x), \tag{23}$$

$$\Delta' = (1/E) \sum_x d'(x) h(x) e(x). \tag{24}$$

4 Inspecting the Propagation Mechanism

Before proceeding to the quantitative part of the paper, here, to build intuition, we study the qualitative properties of the model. We begin by studying the *deterministic* steady-state equilibrium of the model, in which the tax rate is constant over time, and the flows into unemployment (separations) equal the flows out of unemployment (hires). Then, we turn to *perfect foresight* transition dynamics to provide insight into the economy's dynamic response to shocks.

In the model, there are two mechanisms relevant for tax policy:

(i) **Search frictions.** At the individual level, job-meeting probabilities driven by the market tightness ratios determine the *speed* at which unemployed individuals find job opportunities. In this sense, the faster an unemployed bumps into a vacancy, the lesser is the extent of search frictions impinging on equilibrium allocations.

(ii) **Extensive margin of employment adjustment.** At the individual level, the zero-surplus constraint determines whether a worker-employer meeting is converted into a job, or an existing job is destroyed. At the aggregate level, this mechanism determines the mass of *"marginal"* or *"zero-surplus"* workers, a key determinant of the sensitivity of the economy to tax shocks.

To summarize, the dynamic response of the aggregate employment rate to changes in tax rates is driven by two mechanisms. First, the extent of search frictions, as measured by the tightness ratio, varies in response to tax shocks. Notably, the tightness ratio in each submarket, and so the probability that an unemployed worker meets a job vacancy, falls in response to a tax rate hike. Further, because the job-meeting probability is concave in the tightness ratio, it drops more in response to a tax rate hike than to an equally-sized tax rate cut.

Second, shifts in tax rates lead to occasionally-binding zero-surplus constraints, which is a key source of non-linearity in the propagation mechanism of tax shocks. Indeed, the magnitude and persistence of tax shocks hitting the economy impinge on the frequency at which the zero-surplus constraints bind. For example, a tax rate hike can make the zero-surplus constraint bind, implying an immediate adjustment in employment through endogenous job destruction. Further, the larger the tax rate increase, the larger the fraction of submarkets hitting the zero-surplus constraint, the more important is the extensive margin of employment adjustment.

4.1 Deterministic Steady State

Frictional equilibrium From equation (21), the steady-state employment rate in submarket x is

$$e(x) = \frac{h(x)p(\theta(x))}{h(x)p(\theta(x)) + d(x)}. \tag{25}$$

If the steady-state total match surplus is positive, then $h(x) = 1$ and $d(x) = \delta$. In this case, $e(x) = p(\theta(x))/(p(\theta(x)) + \delta)$, as in the standard DMP model. Otherwise, if the steady-state surplus is zero, $h(x) = 0$ and $d(x) = 1$, such that $e(x) = 0$.

Setting $\Gamma(w(x)) = 0$, to simplify calculations, equations (15)-(17) yield the steady-state equations determining total match surplus (26), tightness ratio (27), and wage rate (28), respectively:

$$S(x) = \frac{Ax - B/(1-\tau)}{1 - \beta\left[1 - \delta - \eta p(\theta(x))\right]}, \tag{26}$$

$$k \geq \beta(1-\eta)q(\theta(x))S(x), \tag{27}$$

$$w(x) = (1-\eta)\frac{B}{1-\tau} + \eta\left[Ax + k\theta(x)\right]. \tag{28}$$

The zero-surplus constraint $S(x) = 0$ gives the cutoff on the tax rate $\bar{\tau} = 1 - B/Ax$, such that for $\tau \geq \bar{\tau}$ the match with a worker of skill level x is not viable. Again, the tax rate cutoff is increasing in the skill level, implying that higher skilled workers remain viable for hiring for higher values of the tax rate. (In the stochastic economy, this implies that matches with higher skilled workers are less likely to be destroyed.) Alternatively, one can think in terms of a cutoff on the skill level $\bar{x} = B/(1-\tau)A$, which is increasing in the tax rate, such that workers of skill level $x \leq \bar{x}$ are not viable for employment.

In the steady state with search frictions, the aggregate employment rate is

$$E = \sum_{x>\bar{x}} f(x)e(x) = \sum_{x>\bar{x}} \frac{f(x)p(\theta(x))}{p(\theta(x)) + \delta}, \tag{29}$$

where the extensive margin of employment adjustment is captured by the condition $x > \bar{x}$ in the summation over submarkets, which determines the fraction of the labor force that is viable for employment.

Frictionless equilibrium In the *frictionless* limit, the economy collapses to a collection of *spot* labor markets with indivisible labor and skill heterogeneity. As $k \searrow 0$, $\lim_{k \searrow 0} \theta(x) = \infty$, such that, abusing notation slightly, $p(\infty) = 1$ and $q(\infty) = 0$. In this frictionless economy, the wage in submarket x equals the marginal product of labor, such that $w(x) = Ax$. A reservation wage rule then determines whether an individual is at work, or idle. Notably, the reservation wage is $\bar{w} = B/(1-\tau)$, such that if $w(x) > \bar{w}$, workers of skill level x are employed, otherwise if $w(x) \leq \bar{w}$, they are unemployed or nonemployed.

Finally, setting $\delta = 0$, the aggregate employment rate in the frictionless economy is

$$E = \sum_{x>\bar{x}} f(x) = 1 - F(\bar{x}), \tag{30}$$

where $F(\bar{x})$ is the cumulative distribution function (CDF) of worker skills evaluated at the zero-surplus cutoff $\bar{x}$.

4.2 Convex Hiring Costs

Using the surplus equation (26), the free-entry condition (27), at the interior solution for vacancy posting, and the time discount rate $r \equiv 1/\beta - 1$, we obtain (after some algebra) a relation between net productivity and the tightness ratio, which determines the steady-state equilibrium:

$$Ax - \frac{B}{1-\tau} = \frac{1}{1-\eta}\left[(r+\delta)\frac{k}{q(\theta(x))} + \eta k\theta(x)\right]. \tag{31}$$

On the left-hand side, *net productivity*, which is equal to the marginal product of labor Ax minus the opportunity cost of employment $B/(1-\tau)$, represents the instantaneous return to a match, or the relative return to market versus non-market activity. Permanent shifts in tax rates directly affect this margin, by impinging on work incentives for a given tightness ratio.

The two terms on the right-hand side capture the effect of the tightness ratio on *hiring costs*, a key equilibrium relationship due to the presence of search frictions. The first term, $(r+\delta)k/q(\theta(x))$, is the discounted value of the expected cost of posting and maintaining a vacancy. This term is increasing and concave in market tightness: the higher the tightness ratio, the lower the probability that a vacancy meets an unemployed worker, the higher the duration of an unfilled vacancy. The second term, $\eta k\theta(x)$, captures the positive effect that a tight labor market exerts on the wage. The higher the tightness ratio, the higher the probability an unemployed meets a vacancy, a mechanism that strengthens the worker's bargaining position which puts upward pressure on wages.

Importantly, note that the steady-state employment rate implied by (25) is increasing and concave in tightness, which yields a *convex* relationship between the employment rate and hiring costs. In this sense, search frictions can induce a high degree of curvature in hiring costs, and thereby in the cost of producing output.

4.2.1 Role of Search Frictions

To isolate the role of search frictions from the extensive margin of employment adjustment, here we focus on the case of positive match surplus in which workers' skill levels are above the skill cutoff, i.e., $x > \bar{x}$, for all values of the tax rate. Figure 1 depicts the steady-state equilibrium implied by equation (31), before and after a tax rate increase, for high-skilled workers generating "large surplus" (panels A and B), and for low-skilled or "small surplus" workers (panels C and D).

Panel A shows the steady-state equilibrium for high-skilled workers determined as the intersection of the net productivity curve (NPC) – the left-hand side of equation (31) – and the hiring cost curve (HCC) – the right-hand side of equation (31). Panel B shows two different equilibria: the initial one depicted in panel A, and the new equilibrium that results from an increase in the tax rate. The HCC remains unchanged, whereas the NPC shifts downward, implying a lower employment rate. Similarly, panels C and D illustrate the two equilibria before and after an equally-sized tax rate increase for low-skilled, small-surplus workers. Two main theoretical insights emerge.

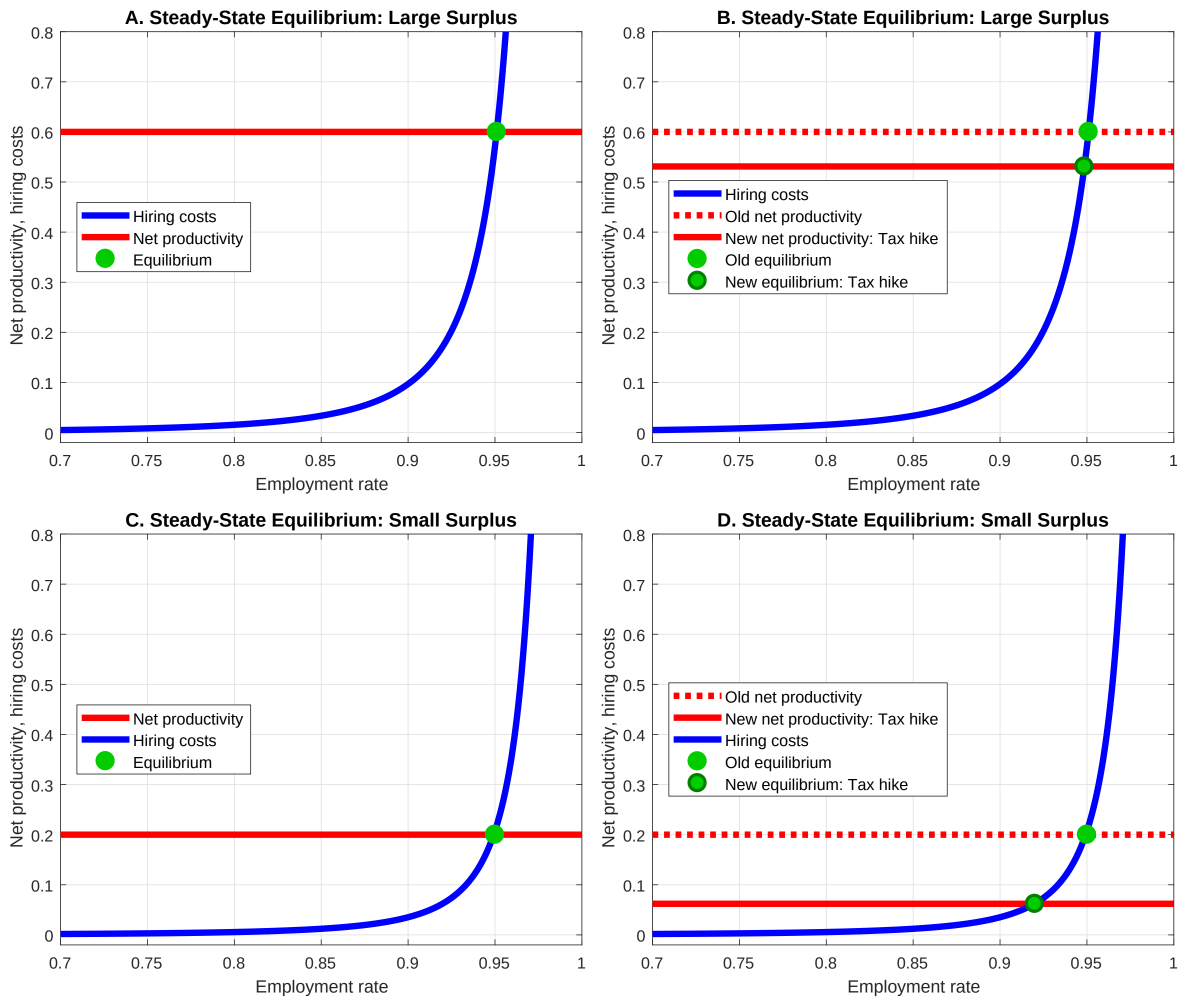

Figure 1: Role of Search Frictions

Notes: The figure illustrates the steady-state equilibrium of the model before and after a tax rate increase. On the x-axis, employment is steady-state employment in equation (25). On the y-axis, net productivity (NP) and hiring costs (HC) are the left- and the right-hand side of equation (31), respectively. Panels A and B refer to high-skilled (large surplus) workers. Panels C and D refer to low-skilled (small surplus) workers. In all panels, worker skill levels are above the skill cutoff for zero surplus.

First, for large-surplus workers, the effect of a tax rate change is generally small, and approximately linear. Second, for small-surplus workers, instead, a tax rate increase of the same size, has a bigger negative effect on the employment rate. Further, the effects of tax rate changes on small-surplus workers are potentially asymmetric. Because of the curvature in HCC, the effect of a tax rate increase can be considerably larger than a tax rate cut. Hence, the overall effect of a tax rate hike or cut on the aggregate employment rate critically depends on the share of low-skilled, small-surplus workers in the unemployment pool. As the composition of the unemployment pool varies over the business cycle, tax rate changes can have state-dependent effects.

4.2.2 Role of Zero-Surplus Constraint

As in Figure 1, panels A and B of Figure 2 depict the equilibrium before and after the increase in the tax rate for high-skilled workers whose surplus remains positive in spite of the tax rate hike. For these workers, the drop in the employment rate induced by the tax rate hike comes entirely from search frictions as in the DMP framework. A tax rate increase depresses vacancy posting, which in turn lowers the probability of finding a job.

Panels C and D of Figure 2 illustrate the effect of the tax rate increase for low-skilled workers, whose match surplus hits the zero-surplus constraint. In this latter case, the employment rate drops to zero. Importantly, the larger the size of the tax rate increase, the larger the share of workers hitting the zero-surplus constraint. This mechanism is an important driver of state-dependence in tax policy. To see this, think of a scenario in which the economy is in a recessionary state in which aggregate productivity $A < \bar{A}$ is below normal, say, $\bar{A}$. In such a recession, NPC has shifted downward (as in the case of a tax rate increase), such that the zero-surplus constraint could be binding for a large share of workers. In this situation, a tax rate cut has a larger effect than it would have otherwise had if aggregate productivity was above normal.

Note also that the stimulative, positive effect of a tax rate cut on employment is dampened by the presence of search frictions. In the absence of frictions, the submarket would jump back up to full employment insofar as the tax rate cut is large enough to make workers viable for hiring again.

4.3 Transition Dynamics

The equilibrium of the model features transition dynamics of different shapes depending on the importance of search frictions vis-à-vis the extensive margin of employment. In the frictionless case, there is no internal propagation of shocks. For example, in response to a tax rate increase followed by an equally-sized decrease, the employment rate falls

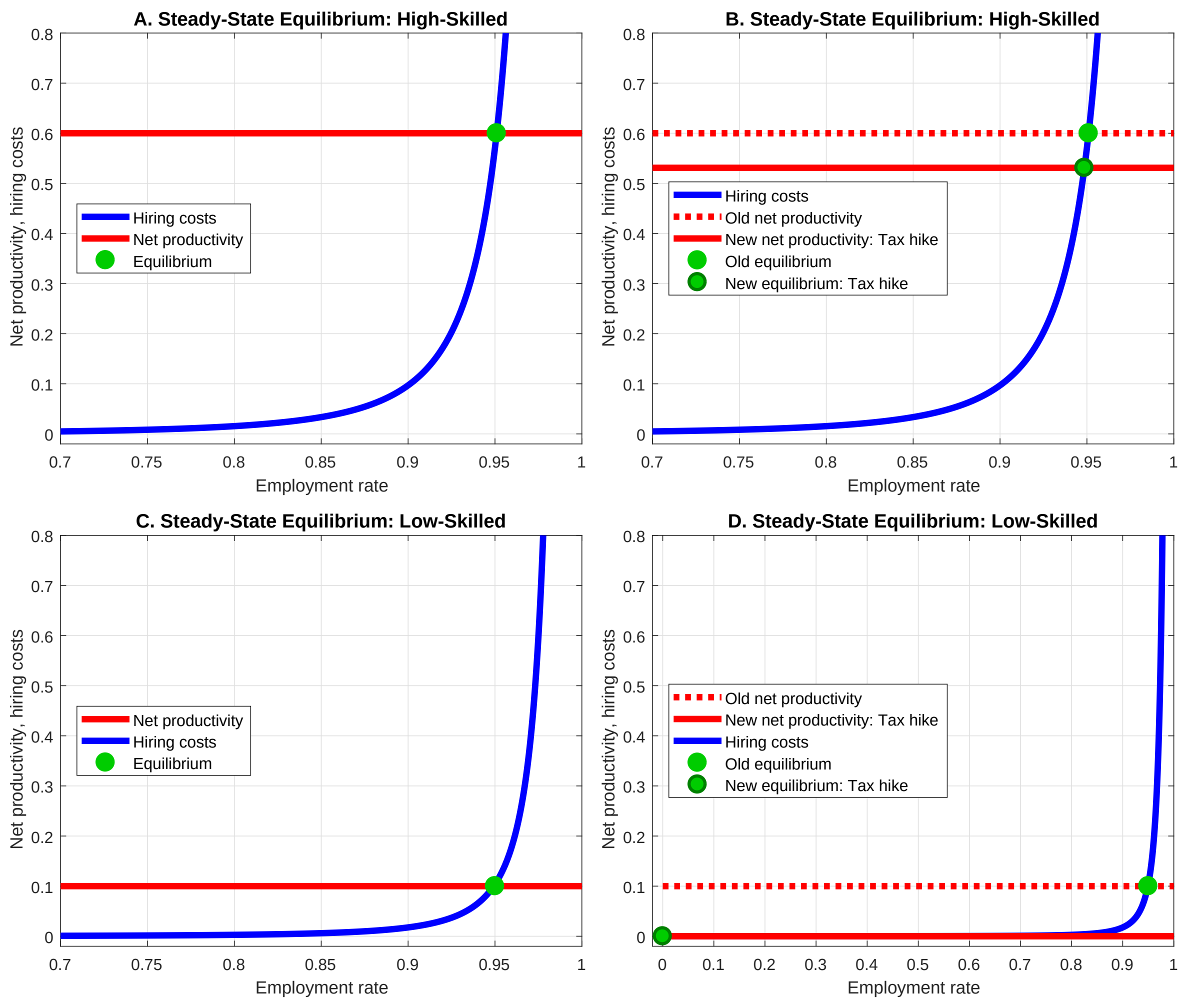

Figure 2: Role of Zero-Surplus Constraint

Notes: The figure illustrates the steady-state equilibrium of the model before and after a tax rate increase. On the x-axis, employment is steady-state employment in equation (25). On the y-axis, net productivity (NP) and hiring costs (HC) are the left- and the right-hand side of equation (31), respectively. High- and low-skilled are defined as workers whose skill levels are above and below the skill cutoff for zero surplus after the tax rate increase.

on impact and then instantaneously jumps back up to its initial level before the shock. By contrast, in the presence of search frictions, and occasionally-binding zero-surplus constraints, transition dynamics is sluggish. In general, employment rate responses to tax rate shocks can take different shapes in between the V- and L-shaped case.

Before proceeding, we note that in the stochastic version of the model, the transition dynamics of the employment rate in submarket x is governed by a stochastic difference equation:

$$e'(x) = \left[1 - h(\tau'; x) p\left(\theta(\tau; x)\right) - d(\tau'; x)\right] e(x) + h(\tau'; x) p\left(\theta(\tau; x)\right). \tag{32}$$

In Section 7, we leverage the full stochastic structure of the model, i.e., model solution and the stochastic difference equation (32) for each submarket, to simulate non-linear IRFs. Here, however, to provide analytical insight, we consider the case of *perfect foresight* transition dynamics with permanent and temporary tax rate shocks.

Permanent tax rate shocks For the case of permanent shocks, think of the following scenario. At time $t = 0$, submarket x rests at the steady-state employment rate $e_0(x) \geq 0$ associated with the tax rate τ_0. Unexpectedly, a permanent tax rate shock realizes. After the shock, the agents expect the tax rate to be $\tau^* \neq \tau_0$ for all $t \geq 1$. Note that since the tightness ratio is a "jump" variable, and the indicator variables for endogenous job creation and destruction depend solely on the value of the tax rate, they jump to their new steady-state values on impact and remain constant thereafter.

Temporary tax rate shocks For the case of temporary shocks, we consider a sequence of so-called "MIT shocks." Again, think of submarket x at the steady state with $e_0(x) \geq 0$. Unexpectedly, a shock realizes, such that agents expect the tax rate to be $\tau^* \neq \tau_0$ for $t \geq 1$. However, after a number of periods, unexpectedly, at time $t = T \geq 2$, a new shock realizes, such that the tax rate takes on a new value, say, τ_0, which is expected to last forever for $t \geq T$. In a nutshell, the agents in the model perceive every shock as permanent, akin to a random walk. However, differently from a random walk, the agents also expect innovations to the tax rate to be zero at all times. In this sense, in the case of MIT shocks, agents are systematically "surprised" by changes in tax rates.

4.3.1 Search Frictions vs. Extensive Margin of Employment Adjustment

In the case of perfect foresight, the employment rate in each submarket evolves over time according to a deterministic difference equation:

$$e_{t+1}(x) = \left[1 - h(\tau^*; x) p\left(\theta(\tau^*; x)\right) - d(\tau^*; x)\right] e_t(x) + h(\tau^*; x) p\left(\theta(\tau^*; x)\right). \tag{33}$$

The solution to (33) is

$$e_t(x) = e^*(x) + \lambda^t(\tau^*; x) \left[e_0(x) - e^*(x) \right], \tag{34}$$

where $e^*(x)$ is the new steady-state level of the employment rate associated with τ^*, and $\lambda(\tau^*; x)$ governs the speed of transition to the new steady state:

$$\lambda(\tau^*; x) \equiv [1 - h(\tau^*; x) p(\theta(\tau^*; x)) - d(\tau^*; x)] \leq 1. \tag{35}$$

As is evident from (35), the speed of adjustment to the shock, $\lambda(\tau^*; x)$, depends on the specific value of the new tax rate τ^* and it varies across submarkets.

Two polar cases Two cases are of special interest. First, absent labor-market frictions (i.e., $\delta = 0$ and $p(\infty) = 1$), $\lambda(\tau^*; x) = 0$. This implies that the employment rate becomes a jump variable: it falls or rises on impact to its new steady-state level, implying no internal propagation of tax shocks.

Second, in the presence of frictions, the model displays a fundamental asymmetry in the dynamic adjustment to tax rate hikes vis-à-vis tax rate cuts. Specifically, if the new tax rate is larger than the tax rate cutoff for zero surplus ($\tau^* \geq \bar{\tau}$), then $\lambda(\tau^*; x) = 0$, so that the employment rate drops on impact to the new lower level, as in the frictionless case. By contrast, in the case of a tax rate cut (with $\tau^* < \bar{\tau}$), then $\lambda(\tau^*; x) = [1 - p(\theta(\tau^*; x)) - \delta] \in (0, 1)$, implying a slow adjustment towards a higher level of the employment rate.

Figure 3 illustrates the variety of responses to tax rate changes implied by the model. In sum, the sign as well as the size of tax rate changes, and the mix of frictionless as opposed to frictional adjustment to shocks, allow for substantial non-linearities and a rich set of IRFs' shapes.

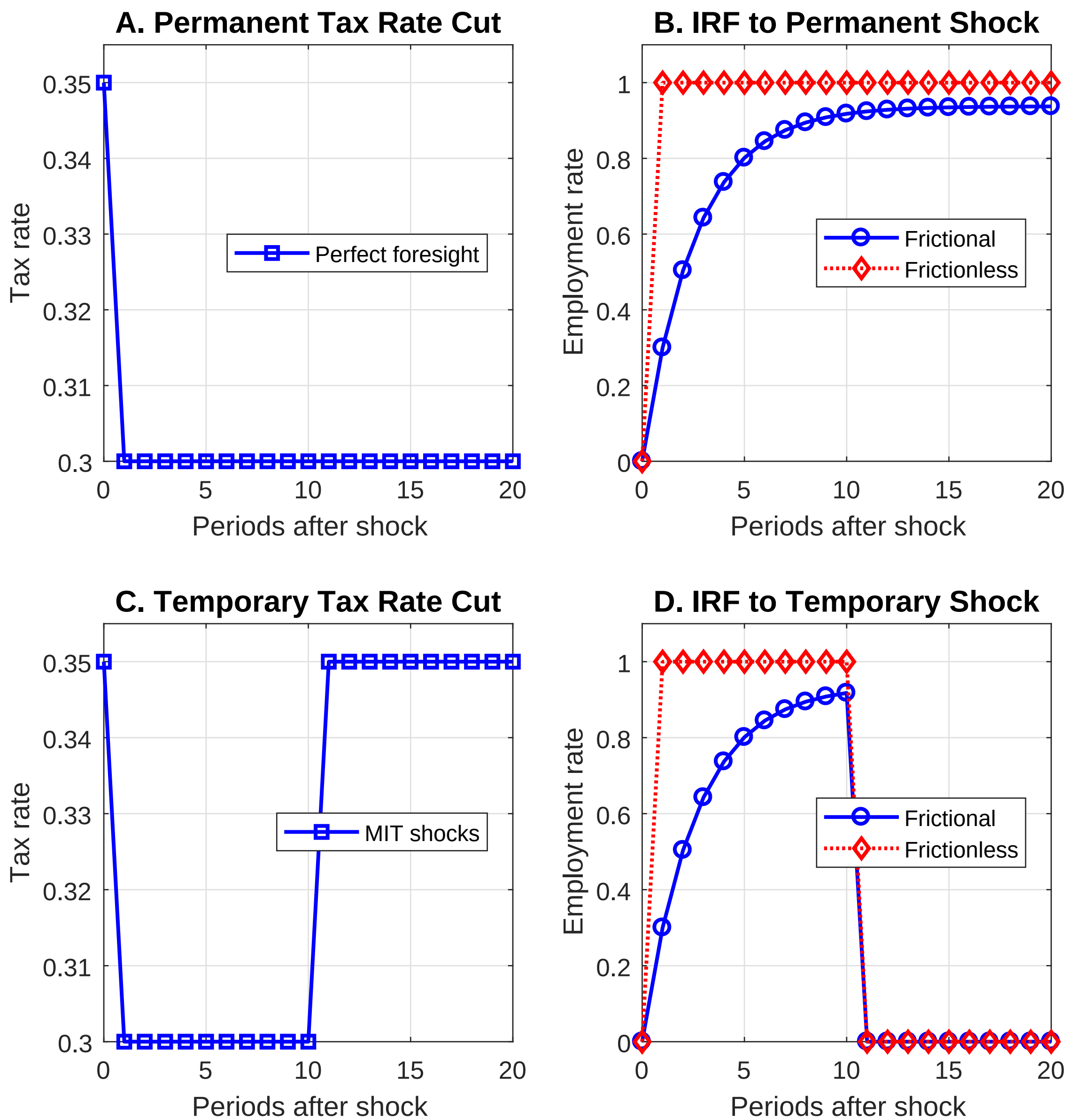

Figure 3: Frictional vs. Frictionless Transition Dynamics

Notes: The figure illustrates the dynamic response of the employment rate in the case with frictions (solid line with circles) and without frictions (dotted line with diamonds) to a permanent tax rate cut (panels A and B), and to a temporary tax rate cut (panels C and D).

4.3.2 Aggregate Dynamics

At the aggregate level, the speed of adjustment of the economy to tax rate shocks is a complex equilibrium object. Specifically, the economy-wide counterpart of (34) is

$$E_t = E^* + \underbrace{\sum_x \lambda^t(\tau^*; x) f(x) \left[\frac{e_0(x) - e^*(x)}{E_0 - E^*} \right]}_{\Lambda_t} (E_0 - E^*), \tag{36}$$

where $E_0 \equiv \sum_x f(x)e_0(x)$ and $E^* \equiv \sum_x f(x)e^*(x)$ are the initial and the new steady-state levels of the aggregate employment rate. Thus, the aggregate speed of adjustment Λ_t depends on: (i) heterogeneous speeds of adjustment across submarket $\lambda(\tau^*; x)$; (ii) different initial employment rates across submarkets $e_0(x)$; as well as (iii) the new steady-state employment rates $e^*(x)$ that vary across submarkets.

5 Bringing the Model to the Data

In this section, we specify the functional form for the meeting technology and discuss parameter values. The parametrization of the model consists of three steps. First, we estimate the parameters governing the stochastic process for tax shocks. Second, to ease comparison with previous work, we calibrate a subset of parameters based on common values in the literature. Third, we estimate the remaining parameters to match a select number of empirical targets, including the *peak* response of the employment rate to a 1 percentage point cut in the average marginal tax rate (AMTR), estimated in the context of proxy-SVARs.[7] See Appendix A for data sources, variables' construction, and estimation details.

5.1 Functional Form for the Meeting Technology

We consider a CRS meeting function, that determines the number of meetings, m, in terms of unemployed workers seeking jobs, u, and job vacancies, v. Based on Den Haan, Ramey

[7] AMTR is the weighted average of the marginal tax rates faced by individual taxpayers. Weights are constructed as individual income divided by total income in a given year. The notion of income includes wages, self-employment, partnership, and S-corporation income. See Barro and Sahasakul (1983) and Barro and Redlick (2011) for additional details.

and Watson (2000), we adopt the following meeting function:

$$m(v,u) = \frac{vu}{\left(v^{\xi} + u^{\xi}\right)^{1/\xi}}, \tag{37}$$

where $\xi > 0$ is a parameter to be calibrated.

With this specification, the probability that an unemployed worker meets a vacancy is $p(\theta) = m(v,u)/u = \left(1+\theta^{-\xi}\right)^{-1/\xi}$, and the probability that a job vacancy meets an unemployed worker is $q(\theta) = m(v,u)/v = \left(1+\theta^{\xi}\right)^{-1/\xi}$. The appealing property of the specification (37) over a Cobb-Douglas is that meeting probabilities are guaranteed to be between 0 and 1. Further, the elasticity of the job-meeting probability with respect to the tightness ratio is $1/(1+\theta^{\xi})$. Differently from a Cobb-Douglas specification, this elasticity is not constant, rather it is decreasing in tightness.

5.2 Parametrization

We now turn to the calibration of the parameters for preferences, technology, and the stochastic process for the tax rate. As standard in a dynamic general equilibrium model like ours, none of the parameters has a one-to-one relationship to a moment. Nonetheless, it is useful to describe the calibration procedure in a few distinct steps.

To summarize, the model has 14 parameters $(\bar{\tau}, \rho, \sigma, \beta, \gamma, \delta, \eta, B, k, \xi, x_{\min}, x_{\max}, \mu_x, \sigma_x)$. A model period is taken to be a "month." Table 1 reports the baseline parameter values. Next, we discuss in detail key steps of the parametrization: data sources, interpretation of model parameters, choice of data moments, and estimation.

5.2.1 Stochastic Process for Tax Shocks

Parametrization of the stochastic process for tax shocks is an important element of our calibration. Our approach, here, is to choose the parameters of the AR(1) process for the tax rate $(\bar{\tau}, \rho, \sigma)$ such that the tax rates in the model reproduce salient properties of AMTRs in the United States. We do so in two steps.

First, we use annual data on AMTRs constructed by Mertens and Montiel Olea (2018) for the period 1946-2012. Then, we estimate an AR(1) process by regressing AMTR on a

Table 1: Baseline Parametrization

Parameter	Description	Value	Source/Target
β	Time discount factor	0.9967	Real interest rate (4%)
γ	UI replacement rate	0.4	Data
δ	Exogenous separation rate	0.02	Data
η	Worker bargaining weight	0.5	Literature
B	Disutility of work	0.1843	Estimated
k	Unit vacancy cost	$[0.011, 3.589]$	Tightness equal 1
ξ	Meeting function	0.89	Estimated
x_{min}	Skills: Lower bound	0.64	Estimated
x_{max}	Skills: Upper bound	10	Exogenously set
μ_x	Skills: Mean	0	Normalization
σ_x	Skills: St. Dev.	0.15	Estimated
$\bar{\tau}$	Tax rate: Mean	0.31	Estimated
ρ	Tax rate: Persistence	0.98	Estimated
σ	Tax rate: Volatility	0.005	Estimated

constant and its lagged value:

$$\text{AMTR}_{t+1} = 0.0183 + 0.9407 \cdot \text{AMTR}_t + \varepsilon_{t+1}^{\text{AMTR}}, \tag{38}$$

where $t = 1946, ..., 2012$ and $\varepsilon_{t+1}^{\text{AMTR}}$ is a residual error term. Our estimates imply a residual standard deviation of 0.0131 and a long-run mean $\bar{\tau}$ of $0.0183/(1 - 0.9407) = 0.3086$. Next, using standard conversion formulas, an autoregressive coefficient of 0.9407 at the annual frequency corresponds to $0.9407^{1/12} = 0.9949$ at the monthly frequency, and an annual standard deviation of 0.0131 corresponds to $0.0131/\sqrt{\sum_{j=1}^{12} 0.9407^{2(j-1)}} = 0.005$ at the monthly frequency.

Second, we discretize the AR(1) process for tax rate shocks (2) using the Rouwenhorst method.[8] Specifically, we use 51 grid points and set the standard deviation σ to 0.005, consistently with the monthly standard deviation calculated above. Setting the persistence parameter ρ to 0.9949 produces, however, an IRF of the tax rate that is too persistent compared to that estimated from SVAR. To overcome this issue, we select the value of ρ such that the IRF of the tax rate to a 1 percentage point shock in the model replicates the

[8] Kopecky and Suen (2010) show that the Rouwenhorst method is more accurate than the other available methods in approximating highly persistent processes.

IRF estimated using narratively-identified shocks to AMTRs in Ferraro and Fiori (2020).

This IRF matching approach is based on an iterative procedure that consists of 5 steps: (i) given the value of $\sigma = 0.005$, guess a value for ρ, say, 0.9949, and approximate the stochastic process for tax rates with the Rouwenhorst method; (ii) simulate artificial time series for tax rates using the Markov chain approximation of the AR(1) process; (iii) fit an AR(1) process to the artificial "monthly" data; (iv) use the estimated autoregressive coefficient to compute an IRF up to 4 years after the shock; (v) iterate until the simulated IRF is close to the estimated IRF. This procedure gives $\rho = 0.98$.

5.2.2 Exogenously Set Parameters

We set the time discount factor β to $(1/1.04)^{1/12} = 0.9967$, such that the model generates an annual real interest rate of 4%, a value that is commonly used in the business cycle literature (see, e.g., McGrattan and Prescott, 2003; Gomme, Ravikumar and Rupert, 2011). We assume that UI benefits $\Gamma(w) = \gamma w$ represent 40% of wages as in Shimer (2005), so we set the replacement rate γ to 0.4.

Next, we turn to three parameters related to labor-market frictions (δ, η, k). We set the exogenous job-separation rate δ to 0.02, consistently with the average of the monthly quit rate in the U.S. nonfarm business sector for 2000:M12-2020:M1. We set the worker bargaining weight η to 0.5. In the median state, i.e., $\tau = \bar{\tau} = 0.31$, the worker's share of surplus is $\eta(1 - \bar{\tau}) = 0.5(1 - 0.31) = 0.345$, that lies between the value of 0.5 in Mortensen and Pissarides (1994) and Hall and Milgrom (2008) and of 0.19 implied by the calibration in Hagedorn and Manovskii (2008).

Finally, we allow for the parameter governing the unit vacancy cost k to vary across submarkets. Similarly to the calibration in Shimer (2005), we set k such that the tightness ratio in each submarket equals one when the tax rate is at the median state. This approach yields that the unit cost of posting a vacancy is increasing in skills.[9] And that the ratio of the unit vacancy cost to skills is increasing and concave in the level of skills. Figure 4 shows the distribution of skills, and the relation between vacancy costs and skills in the calibrated model. Notably, for the highest-skilled workers, vacancy costs represent

[9]This is broadly consistent with the empirical evidence discussed in Hamermesh and Pfann (1996, p. 1268): "The average cost of [employment] adjustment rises very rapidly with the skill of the worker. Thus while external costs may be very low in jobs filled by high-turnover, low-skilled workers, they are very large for high-skilled jobs that are usually occupied by long-tenure workers."

35% of the monthly output of a job, whereas for the lowest-skilled in the labor force, unit vacancy costs are a trivial fraction of output. In this sense, search frictions are relatively more important for the high- than the low-skilled workers.

5.2.3 Jointly Estimated Parameters

Given the exogenously set parameters, and the parametrized stochastic process for the tax rate, we are left to determine 7 parameter values $(B, k, \xi, x_{\min}, x_{\max}, \mu_x, \sigma_x)$. Here, we begin by discussing key model statistics, then we turn to the choice of data moments and estimation.

Opportunity cost of employment The disutility of work B is a key parameter governing willingness to work. At the individual level, the flow opportunity cost of moving from unemployment to employment consists of forgone UI benefits plus the forgone value of leisure. At the aggregate level, the *average* flow opportunity cost of employment is

$$z \equiv \frac{1}{U} \sum_x \left[B + \gamma w(x)\right] u(x), \tag{39}$$

where $u(x)$ is the mass of unemployed in submarket x and $U = \sum_x u(x)$ is aggregate unemployment. Similarly, the aggregate wage rate is calculated as

$$w = \frac{1}{E} \sum_x w(x) e(x), \tag{40}$$

where $e(x)$ is the mass of employed workers in submarket x and $E = \sum_x e(x)$ is aggregate employment. The ratio of z to the *after-tax* wage $(1 - \tau)w$ is akin to a "replacement ratio," which captures the relative return of non-market to market work.

Skill distribution Worker skills $x \in \{x_1, x_2, \ldots, x_M\}$ are distributed according to a Log-Normal $(\mu_x,\ \sigma_x^2)$, where the parameters $-\infty < \mu_x < +\infty$ and $\sigma_x > 0$ govern the scale and the shape of the distribution, respectively.[10] To proceed, we discretize the support of this distribution with a 200-point approximation, where the skill types are evenly spaced over a grid ranging from the lowest $x_{\min}$ to the highest type $x_{\max}$. We normalize $\mu_x = 0$,

[10]This assumption is informed by the evidence on the log-normality of empirical wage distributions (see, e.g., Moscarini, 2005; Pizzinelli, Theodoridis and Zanetti, 2018).

set $x_{\max} = 10$, and calibrate $x_{\min}$ and σ_x jointly with the utility parameter B and the matching function parameter ξ using a simulated method-of-moments procedure, whose implementation we describe below.

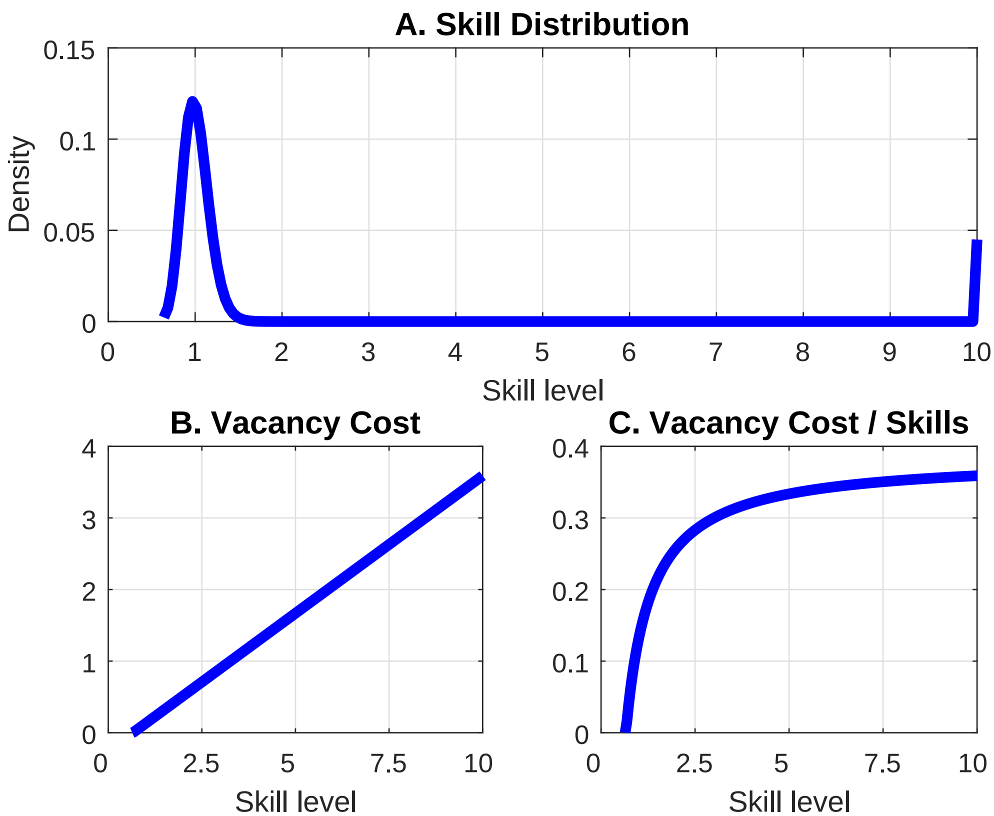

Figure 4: Worker Heterogeneity and Vacancy Costs

Notes: Panel A shows the distribution of skills implied by the calibration of the model: $x_{\min} = 0.64$, $x_{\max} = 10$, $\mu_x = 0$, and $\sigma_x = 0.15$. Panel B shows the values of the vacancy cost parameter k by skill levels. Panel C shows the ratio of the unit vacancy cost to the skill level, i.e., k/x.

Choice of data moments To determine the remaining 4 parameters $(B, \xi, x_{\min}, \sigma_x)$, we estimate their values such that model-implied moments match their corresponding data moments:

1. Average unemployment rate (5.5%);

2. Average job-finding rate (40%);

3. Average opportunity cost of employment relative to after-tax wages (0.7);

4. Peak response of the aggregate employment rate to a 1 ppt cut in the AMTR (0.65 ppt).

A few remarks about the choice of the data moments above are in order. First, the two targets for the average unemployment rate (5.5%) and job-finding rate (40%) are values commonly used in the literature. Depending on different data sources and estimation techniques, available estimates for the average job-finding rate in the United States range from 35% to 45% (see Shimer, 2005, 2012; Fujita and Ramey, 2009). Here we choose the midpoint of the available range of estimates.

Second, there is an ongoing discussion in the literature about the calibrated value of the opportunity cost of employment, or flow value of non-market work. In the standard DMP model, the value of non-market work is a key parameter determining the elasticity of the tightness ratio to productivity as well as to tax shocks. For productivity shocks, Shimer (2005) shows that a standard DMP model, calibrated to a 40% UI replacement rate, fails to generate a realistic elasticity of the unemployment rate with respect to the observed movements in labor productivity. Hagedorn and Manovskii (2008) argue that the flow value of non-market work includes not only UI benefits, but also the forgone values of home production and leisure, and that a calibration based on a replacement ratio as high as 90% improves the model's ability to yield fluctuations in the unemployment rate of a magnitude comparable to the data. Here we choose to target an intermediate value of 0.7 which is consistent with the estimate of Hall and Milgrom (2008).

Third, a peak employment rate response of 0.65 percentage points is within the range of available empirical estimates in the literature. Specifically, while the point estimate is taken from Ferraro and Fiori (2020), the magnitude of such peak response is consistent with existing empirical studies, e.g., Mertens and Montiel Olea (2018). Indeed, estimates based on narratively-identified tax shocks have stood the test of time, by consistently producing IRFs of similar magnitude (see Ramey, 2016, for a survey of the literature).

Simulated method of moments Because of non-linearities, the ergodic distribution of the stochastic economy is not centered around the deterministic steady state. As a result, the average values of endogenous variables differ from their corresponding steady-state values implied in a version of the model with perfect foresight. Moreover, the larger the extent of non-linearities, the larger this discrepancy.

Implementing our method-of-moments approach requires then an iterative procedure. Given an initial guess for parameter values, we solve the equilibrium of the stochastic model, compute the theoretical means of the endogenous variables using the ergodic probability distribution for the tax rate, and compare those theoretical means with the corresponding sample averages. This procedure gives $B = 0.1843$, $\xi = 0.89$, $x_{\text{min}} = 0.64$, and $\sigma_x = 0.15$.

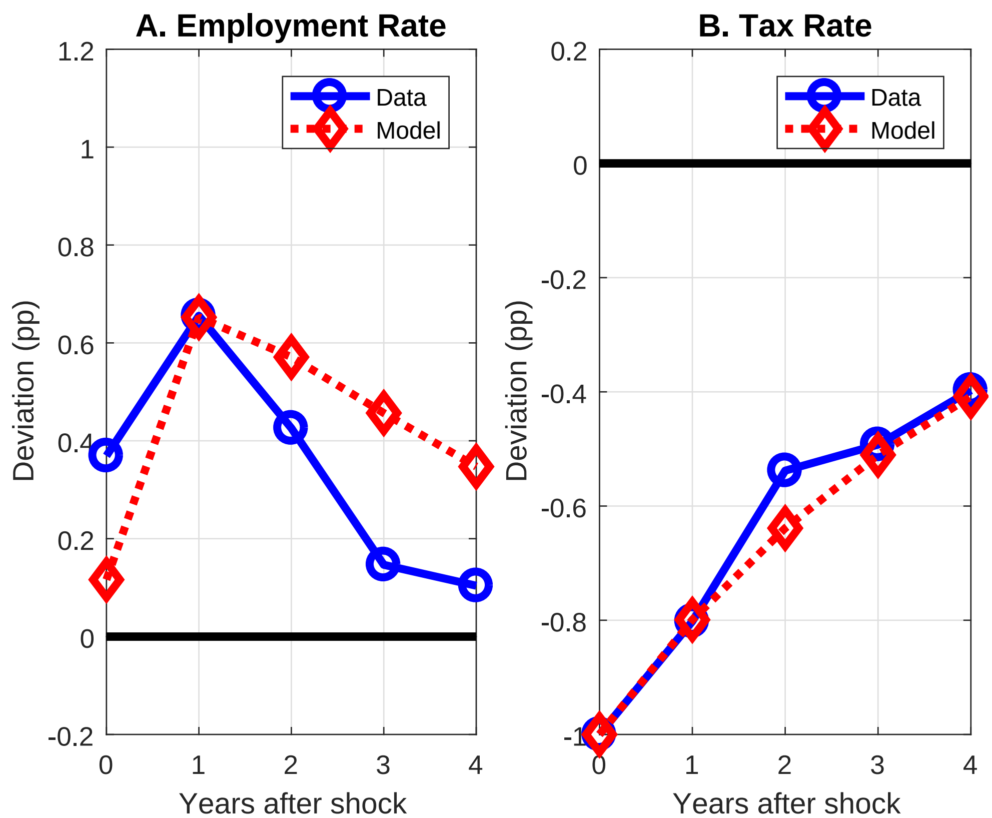

Figure 5: Linear IRF to a Tax Shock - Model vs. Data

Notes: The figure shows the linear IRF estimated on actual data with a proxy SVAR (solid line with circles) and that estimated on artificial data simulated from the model (dotted line with triangles).

5.3 Linear IRFs: Model vs. Data

Figure 5 shows *linear* IRFs to a 1 percentage point cut in the tax rate estimated on artificial data generated from the model (dotted lines with triangles) and from actual data in a proxy-SVAR framework (solid line with circles), up to four years after the shock.

In the model, as in the data, tax rate shocks are persistent with a half-life of roughly 3 years. In fact, the persistence of tax shocks is an important factor determining forward-looking behavior in the model, such as vacancy posting through entry decisions, as well as job destruction and job acceptance decisions. Overall, the model does a good job of accounting for the empirical IRF to the narratively-identified tax rate shock. While, as per our calibration strategy, the model matches the peak response of the employment rate, it generates an IRF that exhibits more persistence relative to the empirical IRF.

Since the IRFs in Figure 5 are linear IRFs, they are the mirror image of the IRFs to an equally-sized tax rate increase. Further, as a by-product of the linearity assumption, the magnitude of the responses, as measured by, say, the peak or trough, as well as the shape, are the same in recessions and expansions. To be sure, linear IRFs are not typically viewed as an *exact* representation of macroeconomic variables, however, a long tradition in the literature takes them as a *"good approximation"* of aggregate dynamics. In that view, linear IRFs are interpreted as weighted averages of the responses to positive and negative shocks, small and big shocks, as well as of the responses in recessions and expansions.

In Sections 6 and 7, we show that the interaction of search frictions with the extensive margin of employment adjustment induces a high degree of non-linearity: a linear IRF would mask quantitatively sizable differences in the aggregate response of the economy to tax shocks based on the state of the business cycle, sign, and size of the shock. For example, the *average* response to a shock is *not* informative about the expected response of the economy to a tax policy change during a recession.

6 Basic Properties of the Calibrated Economy

In this section, we examine basic properties of the calibrated economy, relating tax rates to key aggregate variables, such as the unemployment rate, output, job-finding and job-separation rates.

6.1 Stochastic Steady State

We consider the stochastic steady state of the model, defined as the equilibrium in which endogenous variables remain constant in the presence of expected future shocks, when the innovations to these shocks turn out to be zero.

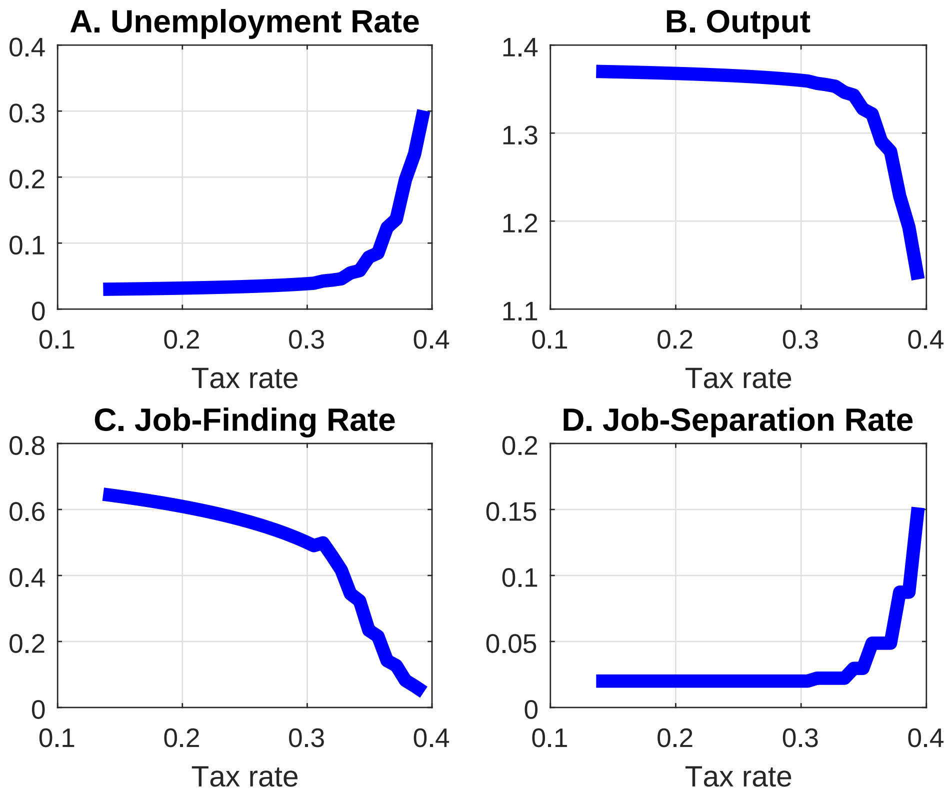

Figure 6: Stochastic Steady States

Notes: The figure shows the stochastic steady state implied by the calibrated model. For each value of the tax rate on the x-axis, panels A through D plot the corresponding values of the aggregate unemployment rate, output, job-finding and job separation rate on the y-axis.

In Figure 6, panels A through D plot the aggregate unemployment rate, output, job-finding and job-separation rates against the tax rate.[11] The unemployment rate is highly non-linear and *convex* in the tax rate. As a result of this convexity, the equilibrium of the model features positive skewness in the unemployment rate, so that the mean of the unemployment rate is above the median. Specifically, in our calibrated economy, the median unemployment rate is 4.4%, as opposed to a mean of 5.5%.

Job-finding and job-separation rates are respectively concave and convex in the tax rate. Again, tax rates can affect job-finding rates through changes in tightness ratios, and

[11]The values of these endogenous variables are constructed assuming that each submarket rests at its own steady state for a given value of the tax rate, and that such tax rate while expected to change over time, it takes on the same value on realization forever.

thereby in the extent of search frictions, and changes in the fraction of marginal or zero-surplus workers that are viable for hiring. In the calibrated economy, median and mean job-finding rates are 0.46 and 0.4, respectively. Similarly, the median job-separation rate is 2.2%, its mean is 2.6%, which is consistent with the estimate in Shimer (2005).

Aggregate output is highly non-linear and *concave* in the tax rate. For a large range of tax rates, output is indeed flat. However, for tax rates above 35%, aggregate output falls sharply. In this sense, the model here displays a property similar to the "plucking model" of fluctuations proposed by Milton Friedman (Friedman, 1993). That is, output features a ceiling, punctuated by sharp contractions induced by sizable increases in tax rates.

6.2 Long-Run Behavior of the Economy

Figures 7 and 8 show (i) the ergodic distribution of the unemployment rate and of the tax rate computed by using 1,000,000 monthly periods simulated from the model, and (ii) the relation between the unemployment rate and the tax rate implied by the ergodic distribution of the model.

Two main results stand out. First, in panel A of Figure 7, the ergodic distribution of the unemployment rate is skewed to the right. Notably, the unemployment rate fluctuates most of the time in the range 4%-6%, with infrequent spikes above the 10% level. In panel B of the same figure, the distribution of tax rates is instead symmetric around the mean. This implies that the skewness in the distribution of unemployment rates is due to the internal propagation mechanism of tax rate changes, not from the stochastic process of tax shocks.

Second, Figure 8 shows that the relation between the unemployment rate and the tax rate from the model's ergodic distribution is convex, which confirms the convexity of the unemployment rate in the stochastic steady state shown in panel A of Figure 6. A new result emerges though: the level of the tax rate determines not only the average unemployment rate, but also the variability of unemployment rates. Notably, the higher the tax rate, the larger the responsiveness of the unemployment rate to tax rate changes. Thus, the history of past realizations of tax rate shocks, too, is a factor determining the dynamic response to a current tax shock. This is yet another form of non-linearity, which cannot be taken in account in the context of linear IRFs.

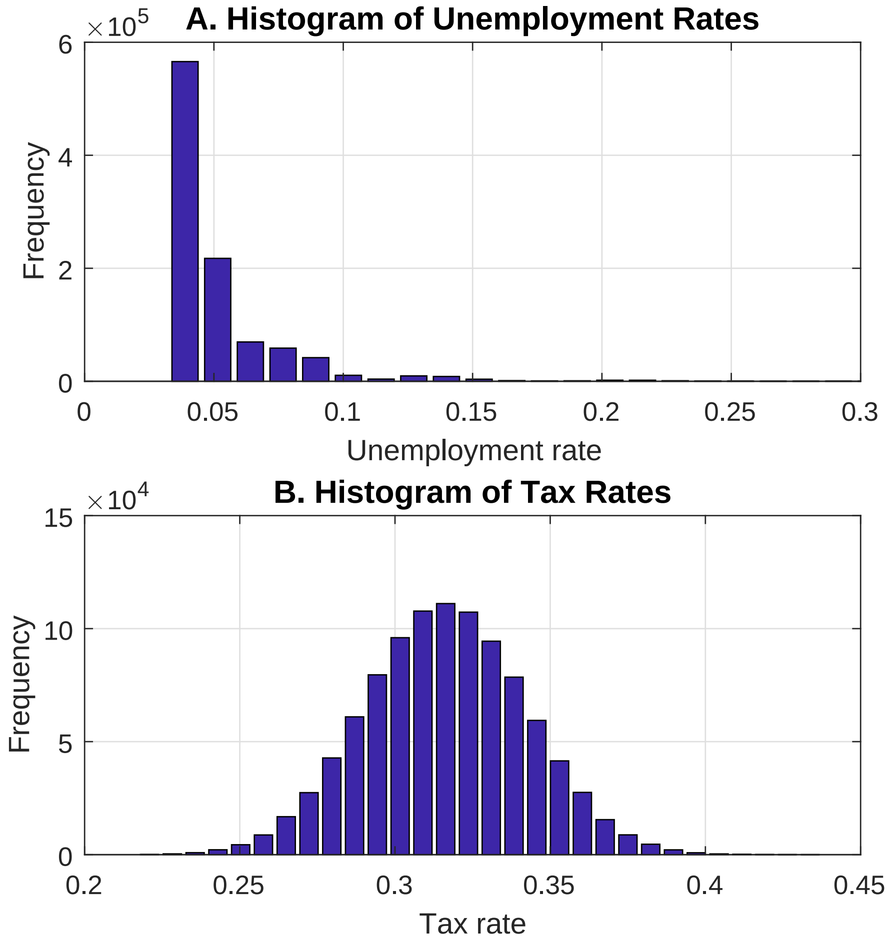

Figure 7: Ergodic Distribution

Notes: The figure shows the histograms of the unemployment rate and tax rate using 1,000,000 monthly periods simulated from the model's ergodic distribution.

7 Non-Linearities in the Propagation of Tax Shocks

To quantify the non-linearities in the propagation mechanism of tax shocks embodied in the model, we rely on *non-linear* IRFs. We focus on tax shocks whose magnitudes are comparable to those observed historically in the United States. In Section 7.1, we briefly discuss issues related to key properties and computation of non-linear IRFs. In Sections 7.2 and 7.3 we report results for asymmetry and state dependence.

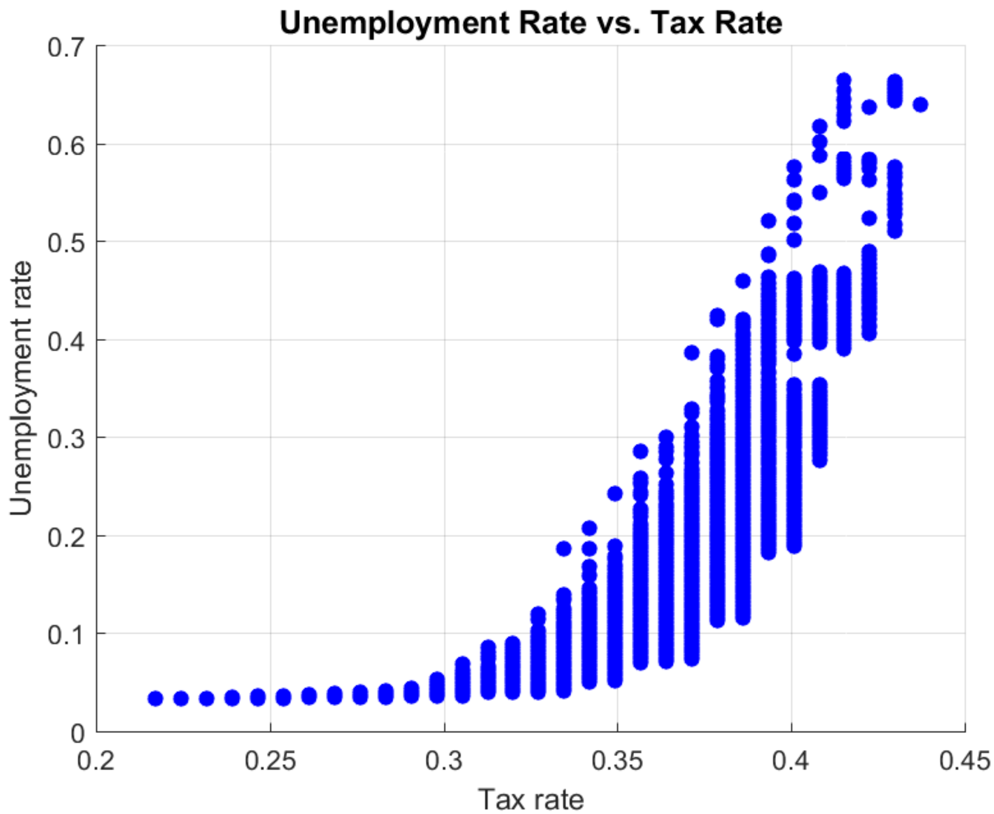

Figure 8: Convex Relation Between Unemployment and Tax Rates

Notes: The figure shows the unemployment rate against the tax rate using 1,000,000 monthly periods simulated from the model's ergodic distribution.

7.1 Non-Linear IRFs

Properties of IRFs In contrast with the more commonly used, linear IRFs, non-linear IRFs are generally *not* invariant to the sign and the size of shocks, nor to the realized sequence of past shocks. As a result, one cannot infer the shape of the IRF to a negative shock from that to a positive shock by simply flipping the sign of the response, nor one can think of an IRF to a small shock as a scaled down version of the response to a big shock. In fact, the magnitude of the marginal effect of a given shock as well as its dynamic implications critically depend on whether the shock is positive or negative, large or small, and whether it was preceded by a history of positive or negative realizations. This creates a well-known reporting problem, that we tackle by producing a number of IRFs under alternative scenarios.

Computation of IRFs In the model, shocks to the tax rate are symmetric, so that the asymmetry in outcomes is solely the result of the internal propagation mechanism at play. Also, the tax rate is the only state variable of the model. This implies that generating time series from the model requires simulating sequences of tax rates, and then using model solutions for individual decision rules and tightness ratios to compute the equilibrium dynamics of the employment rate as implied by the simulated sequences of tax rates.

We compute IRFs by simulating the equilibrium paths of two alternative economies, to which we refer to as "benchmark" and "counterfactual." For the benchmark economy, we simulate 50,000 paths for, say, the employment rate, over 60 model periods ("months"). The sequences of tax shocks are simulated as follows. At $t = 0$, the initial tax rate across all simulations rests at median state, i.e., $\tau_0 = \bar{\tau}$. From $t = 0$ onward, tax rates are simulated according to the Markov chain implied by the Rouwenhorst method. This procedure yields time series of realized tax rates $\left\{\tau_t^i\right\}_{t=0}^{60}$, where $i = 1, 2, \ldots, 50,000$ and $\tau_0^i = \bar{\tau}$ for all replications. Associated to these sequences of tax rates, the model generates 50,000 time series of employment rates $\left\{e_t^i\right\}_{t=0}^{60}$. For the counterfactual economy, we implement the same steps, but with a different initial tax rate $\tau_0 = \bar{\tau} \pm \Delta_\tau$, where Δ_τ parametrizes the size of the initial shock to the tax rate. The average difference between the simulated paths of the benchmark economy and those of the counterfactual economy is the IRF to a $\pm\Delta$ tax rate shock.

To quantify the extent of state dependence, we compare IRFs across different states of the business cycle, as captured by the level of aggregate productivity A. To this goal, we keep the same parameter values as in the baseline calibration, and re-compute the equilibrium of the model with $A = \bar{A} \pm \Delta_A$, where $\pm\Delta_A$ captures the state of the business cycle, relative to a benchmark economy in which aggregate productivity is normalized to $A = \bar{A} \equiv 1$. For example, to compare the IRF to a $-\Delta_\tau$ tax rate cut in "good" versus "bad" times, we compute the IRF to a tax cut in an economy with $A = 1 + \Delta_A$ and the IRF to an equally sized tax cut for an depressed economy in which aggregate productivity $A = 1 - \Delta_A$ is Δ_A percent below normal.

7.2 Asymmetry

Sign asymmetry Figure 9 shows the non-linear IRFs simulated from the model to a 0.75 percentage points tax rate cut (solid line) and to an equally-sized tax rate hike (dotted line). The response of the employment rate to symmetric shocks is asymmetric. Notably, while both responses are hump-shaped, the trough of the IRF to a tax rate hike is larger than the peak response to the equally-sized tax rate cut.

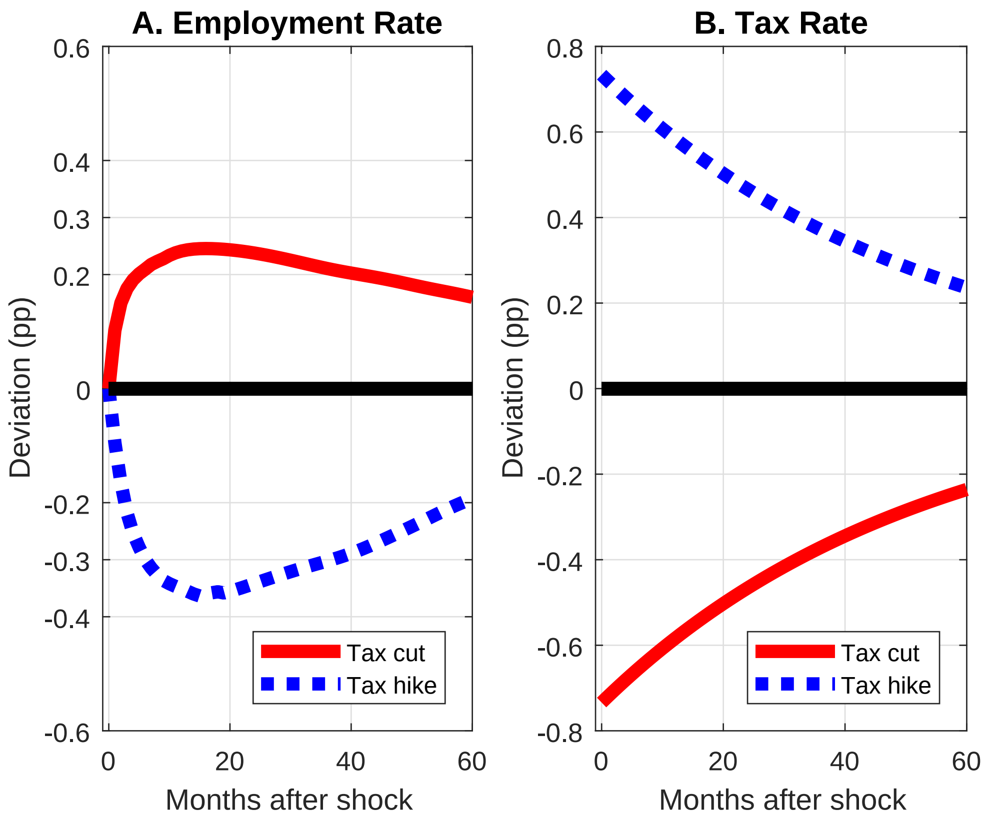

Figure 9: Sign Asymmetry - Small Tax Shock

Figures 10 and 11 show the IRFs to 1.5 and 3 percentage points tax shocks, respectively. In a nutshell, the larger the size of the tax shock, the larger the asymmetry between the response to a tax rate hike and that to a tax rate cut. Next, compare the non-linear IRFs to a tax rate cut in Figures 9-11 with the linear IRF in Figure 5. According to the linear IRF estimated on artificial data generated from the model, at the peak of the response, the employment rate is approximately 0.65 percentage points above the long-run mean. According to the non-linear IRFs simulated from the model, instead, tax rate cuts have much smaller effects on the employment rate. Specifically, a peak response close to 0.65 percentage points can only come from a tax cut of 3, as opposed to 1, percentage points. In sum, the marginal effects implied by the linear IRF in Figure 5 are somewhat similar to those implied by the non-linear IRF to a tax rate hike, but significantly larger than those implied by the non-linear IRF to tax rate cuts.

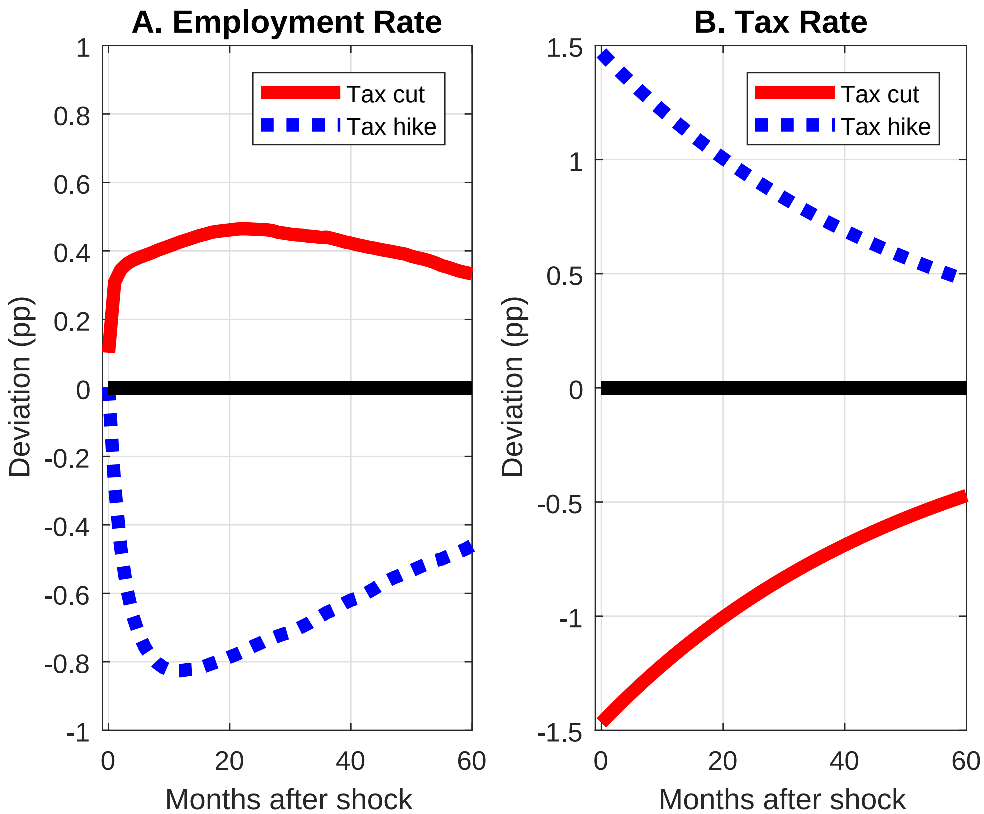

Figure 10: Sign Asymmetry - Medium Tax Shock

Size asymmetry Figure 12 shows the IRF to a 3 percentage points tax rate cut (solid line), and 2 times the IRF to a 1.5 percentage points tax rate cut (dotted line). Similarly, Figure 13 shows IRFs to tax rate hikes. In the context of linear IRFs, the two responses equal each other. Here, instead, IRFs do not scale with the size of the tax shock. Notably, the marginal effect of a tax rate change at each time horizon falls sharply in the size of the shock. Thus, not only the sign, but also the size of the tax shock matters for the shape of the employment rate response. In a nutshell, the larger the size of the tax rate cut, the lesser the effectiveness of tax cuts to increase the employment rate.

7.3 State Dependence

In evaluating the extent of state dependence in the propagation of tax shocks, we consider two scenarios in which aggregate productivity is 4% below (bad times) and 4% above (good times) the level of productivity in normal times $\bar{A} = 1$, thus $A^{\text{good}} = 1 + 0.04$ and $A^{\text{bad}} = 1 - 0.04$.

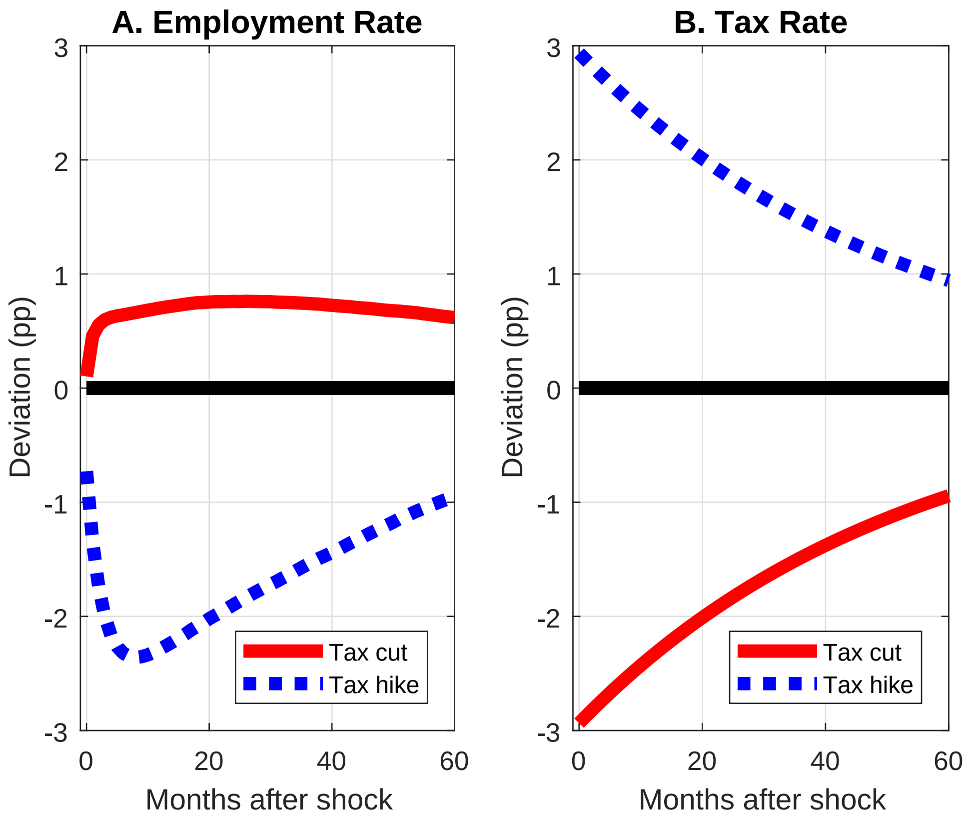

Figure 11: Sign Asymmetry - Large Tax Shock

Figure 14 shows the IRFs to a small tax rate cut in bad times (solid line) and in good times (dotted line). The difference in the magnitude of the responses is sizable. Notably, the increase in the employment rate in bad times is twice as large as that in good times. Importantly, this difference in the responsiveness of the employment rate to tax rate cut becomes larger as we increase the size of the tax rate cut, as shown by Figure 15.

In good times, all unemployment is frictional. Thus, changes in tax rates impinge on equilibrium allocations solely through changes in the extent of search frictions in the labor market. Specifically, a temporary tax rate cut induces an increase in vacancy posting and thereby a higher probability that an unemployment worker meets a posted vacancy. Since in good times, all meetings generate positive surplus, a lower tax rate leads to a higher job-finding probability and so an expansion in employment and output. In bad times, instead, the economy features a mix of frictional and zero-surplus unemployment. In this scenario, a tax rate cut boosts vacancy posting, as in good times; further, it makes zero-surplus workers viable for hiring. Importantly, the deeper the recession, the larger the fraction of zero-surplus workers, the larger the expansionary effect of a tax cut.

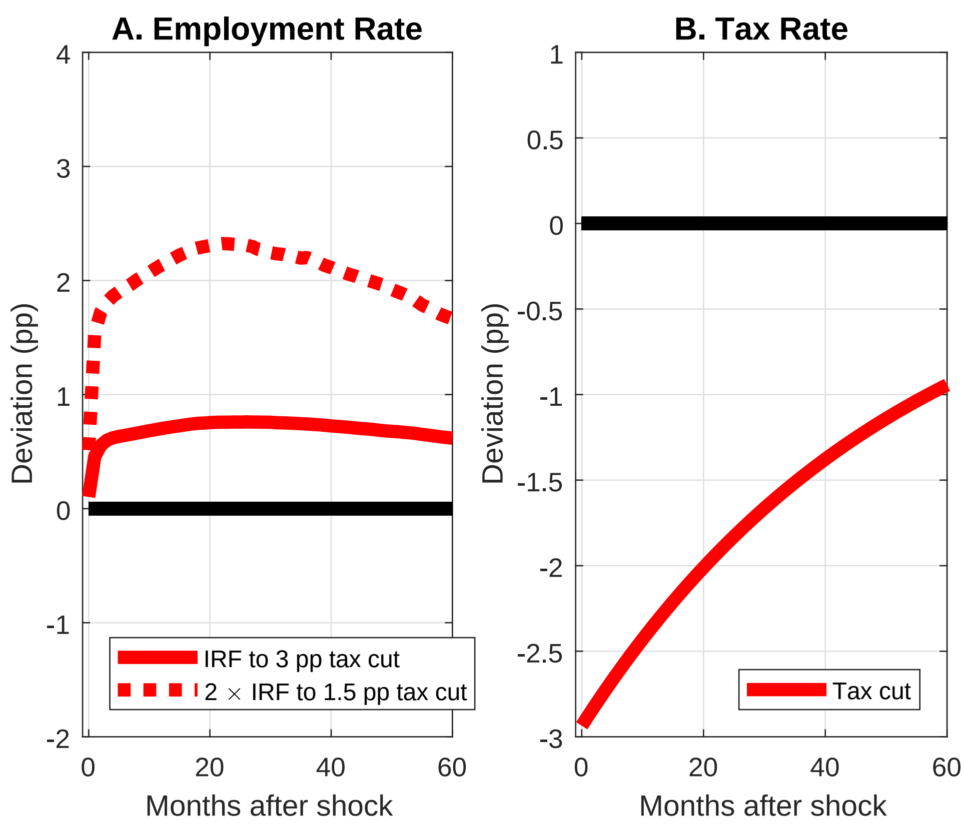

Figure 12: Size Asymmetry - Large Tax Cut

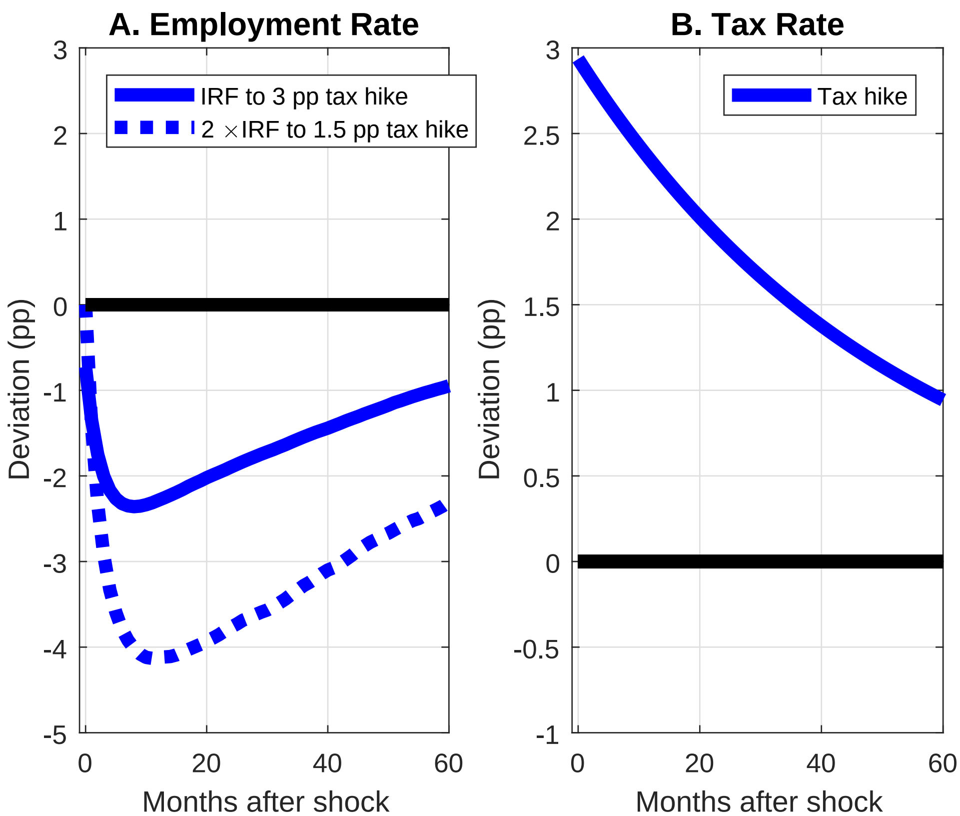

Figure 13: Size Asymmetry - Large Tax Hike

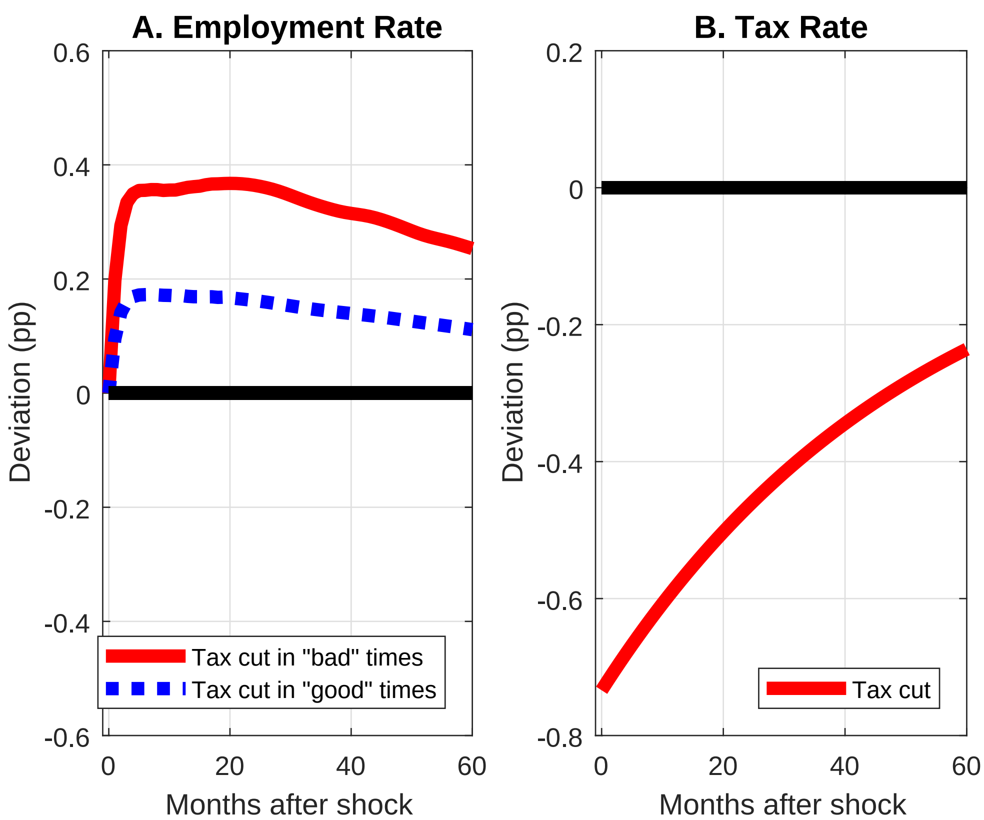

Figure 14: State Dependence - Small Tax Shock

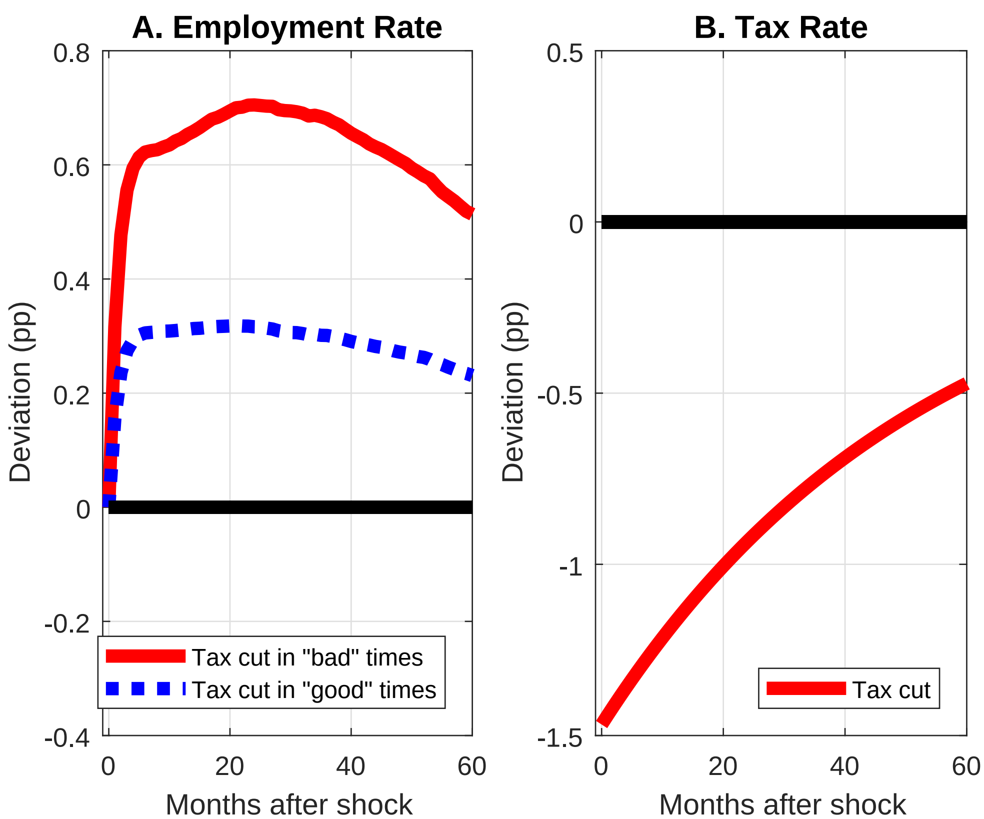

Figure 15: State Dependence - Medium Tax Shock

8 Conclusion

We study the propagation mechanism of tax policy in the context of a heterogeneous-agent equilibrium business cycle model with search frictions in the labor market and an active extensive margin of employment adjustment. The model is estimated to match a select number of data moments for the U.S. economy, including the peak response of the aggregate employment rate to a one percentage point cut in the average marginal tax rate, as estimated in the context of proxy-SVARs based on narratively-identified tax shocks.

In absence of search frictions, the model exhibits a propagation mechanism of tax rate changes that is "approximately linear," such that linear IRFs would accurately capture aggregate equilibrium dynamics. However, in presence of realistic search frictions, and occasionally-binding zero-surplus constraints, we find that the response of the aggregate employment rate to a tax rate change is highly non-linear, displaying asymmetry in the sign and size of the tax rate shock, and state-dependence. Three main results stand out. First, the response to a tax rate cut is considerably smaller than to an equally-sized tax rate hike. Second, the marginal effect of a tax rate cut decreases with the size of the tax cut. Third, the response of the employment rate to a tax rate cut is much larger in a recession than in an expansion.

Overall, the results in this paper suggest that heterogeneity in the composition of the unemployment pool, which varies over the business cycle, and the interaction of search frictions with an extensive margin of employment adjustment can produce significant non-linearities in the propagation mechanism of tax policy. Accounting for these non-linearities seems paramount for the estimation of the effects of countercyclical fiscal policy as well as for the design of automatic stabilizers. While of great importance, we leave these issues for future research.

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Appendix

A Empirics

In this appendix, we describe data sources, variables' construction, and the details of the procedure adopted to estimate the dynamic effects of changes in the average marginal tax rate (AMTR), using a structural vector autoregressions (SVARs) approach, as in Mertens and Montiel Olea (2018) and Ferraro and Fiori (2020).

Average marginal tax rate (AMTR) To construct AMTR, we follow Barro and Redlick (2011) and consider a notion of "labor income" that includes wages, self-employment, partnership, S-corporation income. The data are taken from the CPS March Supplement. AMTR is the sum of the federal individual income tax and the payroll (FICA) tax. We use the NBER-TAXSIM program to simulate marginal income tax rates and marginal payroll tax rates at the individual level. We then construct AMTR as the sum of average marginal individual income tax rate (AMIITR) and average marginal payroll tax rate (AMPTR), using adjusted gross income (AGI) shares as weights.

Identification of tax shocks Tax shocks are identified in the context of SVARs using proxies for exogenous variation in tax rates as external instruments (Mertens and Ravn, 2013). We use the proxies constructed in Ferraro and Fiori (2020) for exogenous changes in AMTRs. To select instances of exogenous variation in tax rates, Ferraro and Fiori (2020) follow the narrative approach proposed by Romer and Romer (2010): changes in total tax liabilities are classified as "exogenous" based on the motivation for the legislative action being either long-run considerations, that are unrelated to the business cycle, or inherited budget deficits.

To account for potential "anticipation effects," only individual income tax liability changes legislated and implemented within the year are included, this approach is in line with Mertens and Montiel Olea (2018). According to this criterion, seven tax reforms are identified as exogenous: (1) Revenue Act of 1964; (2) Revenue Act of 1978; (3) Economic Recovery Tax Act 1981; (4) Tax Reform Act of 1986; (5) Omnibus Budget Reconciliation Act of 1990; (6) Omnibus Budget Reconciliation Act of 1993; (7) Jobs and Growth Tax

Relief Reconciliation Act of 2003.

The impact of a reform is measured as the difference between two counterfactual tax rates. The first counterfactual tax rate is calculated using year $t-1$ income distribution and year t statutory tax rates and brackets. The second is calculated based on the year $t-1$ income distribution and year $t-1$ statutory tax rates and brackets. The difference between the two isolates then the impact that a tax reform implemented in year t had on the AMTR. An issue that arises with these type of calculations is the indexing of the federal tax system starting in 1985. To address this concern, we rescale incomes by the automatic adjustments in bracket widths embedded in the federal tax code.

SVAR specification The baseline reduced-form VAR specification includes the average marginal tax rate, the unemployment rate, the participation rate, and a set of aggregate control variables for the sample of annual observations for the period 1961-2012. Control variables include the log of real GDP per capita, the log of the S&P index, and the federal funds rate, which allows us to capture business cycle dynamics, the monetary policy stance, as well as the effects of bracket creep. To explicitly allow for the feedback from debt to taxes and spending, the log of real government spending per capita (purchases and net transfers), the average tax rate and the change in log real federal government debt per capita are also included.