Hours and Employment Over the Business Cycle: A Structural Analysis*

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Abstract

We conduct Bayesian inference on a quantitative business-cycle model with search-and-matching frictions and a neoclassical hours-supply decision. Likelihood maximization with both U.S. macroeconomic and labor data shows the model cannot jointly reproduce the co-movement of the labor margins with themselves and with macro data. A parsimonious set of features reconciles the model with the data: non-separable preferences with parametrized wealth effects and costly hours adjustment. The model offers a structural explanation for the observed time-varying comovement between the labor margins, being either positive or negative, across post-war U.S. recessions and recoveries. Moreover, the estimated model shows adjustment in the intensive margin contributes up to half the dynamics of total hours in these episodes, as intensive-margin adjustments increase employment losses during recessions and delay employment recoveries.

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1 Introduction

In many OECD countries, changes in average hours per worker are about as large as changes in employment (Ohanian and Raffo, 2012). For instance, in the U.S. economy, the volatility of the intensive margin (average hours per worker) accounts for approximately one-third of the unconditional variability of aggregate hours, while the remainder is attributed to movements in and out of employment (the extensive margin). Moreover, as documented in the next section of the paper, the two margins covary either positively or negatively in specific business cycle episodes, and their relative contribution to aggregate fluctuations is time-varying. What accounts for fluctuations in the labor margins, their comovement with macroeconomic variables, and their contribution to aggregate dynamics? Answering these questions is paramount for both positive and normative analysis, as the economy’s response to aggregate shocks and macroeconomic policies depends on what drives the labor margins over time.

This paper takes up the challenge of accounting for and explaining the cyclical properties of hours per worker and employment in a quantitative search-and-matching model—the benchmark theory of equilibrium unemployment and the cornerstone of labor-market policy analysis. Our contribution to the literature is twofold. First, using Bayesian methods, we determine under which conditions the model can account jointly for the relation of hours per worker with employment and their comovement with macroeconomic data. Second, we use the estimated model to provide a structural assessment of the intensive margin’s contribution to aggregate fluctuations, shedding new light on the sources of labor market dynamics.

Our baseline specification for the labor market features search-and-matching frictions (Diamond, 1982a, Diamond, 1982b, Mortensen and Pissarides, 1994, and Pissarides, 2000) and adjustment along the intensive margin, as in Andolfatto (1996), Arseneau and Chugh (2008), Christiano et al. (2011), Cooley and Quadrini (1999), Ravenna and Walsh (2012), and Trigari (2009), to name a few. Firms adjust labor inputs either by posting vacancies or changing the number of hours per worker. Households face a standard neoclassical decision, which also is consistent with the business cycle literature. In equilibrium, hours per worker equate the marginal rate of substitution between hours and consumption to the value of the marginal product of hours, i.e., hours maximize the joint surplus of a match. The widespread preference for this setup in the literature stems from its invariance to the Barro (1977) critique, since in this environment wages do not have a direct impact on on-going worker-employer relations (and thus on the adjustment of hours per
To successfully account for key properties of macroeconomic data, we embed this labor market structure in a state-of-the-art, quantitative business cycle model, following Christiano et al. (2005), Smets and Wouters (2007), and Justiniano et al. (2010). The model features standard ingredients from this literature, including habit formation, investment adjustment costs, variable capital utilization, and nominal rigidities.

In addition, we include standard demand and supply shocks, namely exogenous fluctuations in total factor productivity, household preferences, the relative price of investment, firms’ desired mark-ups, and monetary and fiscal policy. We also include two standard labor market shocks—variability in the workers’ bargaining power and preference for leisure, which affects hours’ supply. This broad set of disturbances allows us to encompass most of the views on the sources of business cycles found in the literature, giving disturbances other than neutral technology shocks a fair chance of accounting for labor market adjustments. In addition, it allows us to differentiate competing forces behind the time variation in the labor margins’ comovement. We estimate the model using Bayesian inference on U.S. data, including standard macroeconomic observables, as well as total hours and employment. The full information approach allows us to assess the empirical fit of the model against a large set of macro moments, beyond pure labor market outcomes. It also permits a Bayesian counterfactual experiment that quantifies the historical contribution of the intensive margin’s adjustment to aggregate fluctuations.

Our analysis yields three main results. While a Bayesian prior-predictive analysis shows that the baseline model in principle can account for individual variance shares of the labor margins, i.e., their relative variability or comovement, conditioning the model on our observables results in posterior estimates that imply counterfactual labor-market dynamics. In other words, likelihood maximization with both U.S. macroeconomic and labor data renders structural parameter estimates that cannot jointly reproduce the comovement of the labor margins with themselves and with macro data. In particular, the model cannot reproduce the positive unconditional covariance between employment and hours per worker, and it generates counterfactual volatilities for both labor margins. The model also cannot account for the covariance between both labor margins and macroeconomic times series; namely it generates counterfactually low (high) comovement of hours

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An alternative determination of hours per worker occasionally considered in the literature is right-to-manage. This setup is not immune to Barro’s critique. As discussed in the paper, we find the fit of this specification inferior to the baseline model.

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The macroeconomic series include the growth rates of real output, consumption, investment, and wages, inflation, and the Fed Funds rate. As is common in the literature, we first estimate the baseline model using total hours as the only labor market observable (dropping one labor-market shock). We then estimate an alternative version including both labor market series and shocks.
per worker (employment) with aggregate output and consumption. The main issue is that the model implies a countercyclical behavior of hours per worker conditional on both technology and demand shocks, a result contrary to existing VAR evidence—see for instance, Ravn and Simonelli (2007). Moreover, the intensive margin’s relative volatility is counterfactually high.

Our second result is to reconcile the model with the data. Two candidate explanations for the counterfactual labor market dynamics are the strength of short-run wealth effects on labor supply, which affects the response of hours per worker to TFP and demand shocks, and the asymmetric cost of adjusting the two labor margins, since posting vacancies is costly, while the adjustment in hours per worker is frictionless. For these reasons, we introduce additional flexibility in the model by including two ancillary features: non-separable preferences that feature parametrized wealth effects on labor supply as in Jaimovich and Rebelo (2009) and costly hours’ adjustment.3 We re-estimate the model and find strong support in favor of weak short-run wealth effects and positive hours adjustment costs.4 Intuitively, weakening the wealth effect eliminates the negative comovement between hours per worker and employment and increases the comovement of hours per worker with both output and investment. The presence of costly hours’ adjustment reduces the variability in hours per worker, the second key dimension for reproducing the cyclical behavior of both margins of labor adjustment.

Finally, we use the estimated model to examine the behavior of hours and employment in post-WWII U.S. recessions and recoveries. The estimated model offers a structural interpretation for the observed time-varying comovement between hours per worker and employment in these episodes. Hours and employment comove positively in response to demand and supply shocks, and comove negatively in response to labor-market shocks—shocks that affect the rigidity of real wages and hours supply. Moreover, a Bayesian counterfactual experiment that holds hours per worker constant shows that adjustment in the intensive margin had sizable effects on the dynamics of total hours; for instance, with the intensive margin fixed, the decline in total hours is halved in the 1982 recession. A notable component stems from the indirect effect of hours per worker on employment

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3 The preference specification in Jaimovich and Rebelo (2009) includes the limiting case of no wealth effects considered by Greenwood et al. (1988), while preserving the existence of balanced growth in the model.

4 These results are consistent with Imbens et al. (1999), who provide microeconomic evidence of weak short-run wealth effects. Positive hours adjustment costs reflect the fact that, in the data, the regulation of working time and technological frictions constrain hours adjustment. For instance, the “Wages and the Fair Labor Standards Act” establishes that covered nonexempt employees in both private and public sectors must receive overtime pay at one-half times the regular rate for hours over 40 per week. In addition, the incidence of flexible work hours varies by occupation (Beers, 2000); it is lower for jobs that dictate set intervals for work (e.g., nurses, firefighters, pilots) and is most common among workers in executive, administrative, and sales and managerial occupations. Other technological constraints include set-up costs and coordination issues.
beyond the direct effect of hours per worker on total hours): the intensive-margin adjustment increases employment losses during recessions and delays employment recoveries.

The intuition for these results is the following. With the intensive margin fixed, the expected present discounted value of new matches varies less in response to shocks that induce positive comovement between the margins of labor. Accordingly, firms adjust effective capital more than employment, holding all else constant. Lack of adjustment in hours per worker also implies that firms can no longer substitute across the labor margins in response to labor-market shocks, i.e., shocks that induce a negative comovement between hours and employment. As a result, both effects imply that fluctuations in hours per worker increase the response of employment to aggregate shocks.

Our structural assessment of the role of intensive-margin fluctuations in models with frictional labor markets paves the ground for the design of quantitative search-and-matching models featuring hours-per-worker adjustments for both positive and normative analysis. In addition, while we estimate the model on U.S. data, the results of the paper are broader in scope, as the inability of the baseline model to account for the margins of labor adjustment is not limited to the U.S. economy. For instance, as documented by Ohanian and Raffo (2012), hours and employment positively comove in several economies (for instance, in the U.K. and Canada). In addition, parametrized wealth effects and costly hours’ adjustment introduce enough flexibility to match the broad array of empirical covariances of hours per worker and employment observed in the cross-section of countries. Discerning the role of intensive-margin adjustment for other countries is an important avenue for future research.

**Related Literature** This paper relates to different strands of the literature. One branch addresses the variability in the margins of total hours. Some early contributions, including Cho and Cooley (1994), Kydland and Prescott (1991), and Hansen and Sargent (1988), calibrate models in which the supply of total hours adjusts along both the intensive and extensive margins, but abstracts from search-and-matching frictions. A few more recent contributions cast the analysis within the search-and-matching model. In particular, Cooley and Quadrini (1999) build a monetary search-and-matching model with fluctuations in hours per worker to assess the model’s ability to replicate correlations between employment and inflation. Cooper et al. (2007) study hours, employment, vacancies, and unemployment at the micro and macro levels. To do so, they consider a model with rich firm heterogeneity while abstracting from capital accumulation dynamics.
and labor supply considerations. Finally, Kudoh et al. (2019) build a search-and-matching model where firms have the right-to-manage hours that replicates the relative variability of employment and hours per worker observed in Japan. Relative to these studies, our contribution is threefold. First, using Bayesian methods, we determine under which conditions a quantitative search-and-matching model can account for the behavior of hours per worker and employment, including their comovement with macroeconomic data. Second, we do not restrict the analysis to the unconditional volatility of the margins of labor, but address both their unconditional and conditional variability. Third, we provide a structural assessment of the intensive margin’s contribution to aggregate fluctuations.

As highlighted above, several contributions in the search-and-matching literature (including Fang and Rogerson, 2009, and Kudoh and Sasaki, 2011) model both labor margins, although their focus is not on the quantitative behaviour of the composition of total hours. Moreover, since Shimer (2005), another branch of the literature addresses the ability of the search-and-matching model to replicate the cyclical behavior of vacancies and employment. While the debate has for the most part focused on calibrated versions of the search model, a few recent contributions examine the issue in the context of quantitative, estimated models (e.g., Christiano et al., 2016, Gertler et al., 2008, and Justiniano and Michelacci, 2012). In particular, Gertler et al. (2008) show a model with only the extensive margin is able to reproduce the joint dynamics of employment and macroeconomic variables. However, we show that when the intensive margin is introduced in the model, its ability to account for the correlations between labor market variables and aggregate macroeconomic series is significantly impaired.

Finally, this paper is related to the literature addressing the behavior of employment in U.S. cyclical recoveries. In particular, an active strand of research addresses the so-called “jobless recoveries” following the past three U.S. recessions (1991, 2001, and 2009), where aggregate employment continued to decline for years following the turning point in aggregate income and output. Our results provide additional insights to the debate by emphasizing the role of hours’ adjustment for

\footnote{Christiano et al. (2011) estimate a small-open economy model featuring search-and-matching frictions and endogenous hours per worker. They do not address the model’s ability to match the margins of labor adjustment. Altug et al. (2011) show that financial frictions contribute to the dynamics of employment and hours per worker in a small-open economy model calibrated to match features of emerging economies. Balleer et al. (2016) identify, quantify, and interpret the dynamics of short-time work (i.e., publicly subsidized work time reductions) in Germany.}

\footnote{Some studies attribute the occurrence of this phenomenon to fundamental changes in the underlying economic structure (e.g., Schrefl et al., 2005 and Groshen and Potter, 2003). Others focus on cyclical explanations, such as the intensive margin of labor adjustment in the wake of a short and shallow recession (Bachmann, 2012). Jaimovich and Siu (2012) show that jobless recoveries in the aggregate are accounted for by jobless recoveries in middle-skill occupations. Gali et al. (2012) study slower recoveries in an estimated model that abstracts from endogenous fluctuations in hours per worker.}

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employment dynamics during these episodes.

Outline The rest of the paper is organized as follows. Section 2 reviews the empirical relation of U.S. hours and employment. Section 3 outlines the model. Section 4 describes the approach for inference and discusses the cyclical behavior of the margins of labor adjustment in the estimated baseline model. Section 5 studies the performance of the alternative model featuring parametrized wealth effects and hours adjustment costs. Section 6 discusses the cyclical behavior of hours per worker and employment in post-war U.S. recoveries. Section 7 evaluates the robustness of the results to alternative model specifications. Section 8 concludes.

2 Hours and Employment in the Data

This section presents stylized facts about U.S. hours per worker, employment, and total hours worked. We first provide updated estimates of the contributions of the intensive and extensive margins to the variance of total hours. We then present new evidence on the time-variation in the intensive margin’s contribution to recessions and recoveries, as well as its qualitative comovement with employment.

In contrast to previous work, we use measures of total hours worked and employment for the entire economy from the BLS’ Current Employment Statistics (CES) survey. Francis and Ramey (2009) show this economy-wide total hours series is less sensitive to sectoral shifts than nonfarm business sector measures. We use quarterly data over the period 1965:Q1-2007:Q4, which corresponds to the estimation sample period in Section 4. Appendix A provides details of the data construction and sensitivity analysis with a longer data sample and alternative measures constructed from the Current Population Survey (CPS) or labor market measures used in the estimation literature (e.g., Smets and Wouters, 2007). Over the sample period, employment exhibits an upward trend while hours per worker exhibits a downward trend.\(^7\) We consider several alternative detrending methods. Our preferred approach removes a linear trend from each series, since their sum almost perfectly matches the original, demeaned total hours series in this case (their correlation is over 0.99). Thus, the linear filtering appears to account for the low-frequency structural features of employment and hours per worker while preserving the original properties of the total hours

\(^7\)As shown by Kirkland (2000), the decline in average hours per worker recorded by the CES survey can be attributed to the disproportionate increase of nonsupervisory workers in retail trade and services—the two industry divisions in the service-producing sector with the lowest average weekly hours—together with the decline in the percentage of production workers in mining and manufacturing—the two divisions with the highest number of average weekly hours. See also Wolters (2016) for a discussion of low-frequency movements in hours per worker.
series. In addition, we also consider HP filtering with a smoothing parameter of 1600 or $10^5$ and a band pass filter as in Christiano and Fitzgerald (2003).

Table 1 displays these variance shares for the alternative detrending methods. To assess the contribution of the intensive margin to labor adjustment, we consider two standard decompositions of the variance of total hours. The first decomposition computes the shares of the variance attributed to hours per worker and employment as

$$\beta_{\text{cov}, h} = \frac{\text{cov}(TH_t, h_t)}{\text{var}(TH_t)}$$

and

$$\beta_{\text{cov}, L} = \frac{\text{cov}(TH_t, L_t)}{\text{var}(TH_t)}.$$  

The second decomposition defines the shares of the variance attributed to hours per worker, employment, and the covariance term respectively as

$$\beta_h = \frac{\text{var}(h_t)}{\text{var}(TH_t)}; \quad \beta_L = \frac{\text{var}(L_t)}{\text{var}(TH_t)}; \quad \text{and} \quad \beta_{\text{cov}} = \frac{2\text{cov}(h_t, L_t)}{\text{var}(TH_t)}.$$  

While employment accounts for the largest share of variation in total hours, the intensive margin plays a quantitatively significant role. The first decomposition shows the covariance of hours per worker and total hours can account for up to 30 percent of the unconditional variation in total hours, consistent with previous findings for the U.S. (e.g., Hansen, 1985). The second decomposition shows hours per worker and employment positively co-move. The positive covariance of the intensive and extensive margins is a substantial contributor to the variability of total hours ($20 - 30\%$), on top of the direct share of hours per worker’s variance ($10 - 18\%$). Thus, fluctuations in the intensive margin affect total hours both directly and indirectly through employment.

### Table 1: Total Hours’ Decompositions

<table>
<thead>
<tr>
<th>Filtering</th>
<th>$\beta_{\text{cov}, h}$</th>
<th>$\beta_{\text{cov}, L}$</th>
<th>$\beta_h$</th>
<th>$\beta_L$</th>
<th>$\beta_{\text{cov}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.33</td>
<td>0.67</td>
<td>0.18</td>
<td>0.51</td>
<td>0.31</td>
</tr>
<tr>
<td>HP 1600</td>
<td>0.21</td>
<td>0.79</td>
<td>0.10</td>
<td>0.67</td>
<td>0.23</td>
</tr>
<tr>
<td>HP $10^5$</td>
<td>0.25</td>
<td>0.75</td>
<td>0.10</td>
<td>0.60</td>
<td>0.30</td>
</tr>
<tr>
<td>BP</td>
<td>0.23</td>
<td>0.77</td>
<td>0.10</td>
<td>0.63</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Total hours ($TH_t$) and employment ($L_t$) are divided by the civilian non-institutional population to express in per capita terms. Hours per worker ($h_t$) is constructed from $TH_t$ and $L_t$ series. All variables are expressed in logs and multiplied by 100.

While Table 1 confirms the literature’s previous finding on the unconditional contribution of hours per worker and employment to total hours, we highlight an additional empirical regularity: the comovement varies qualitatively in specific episodes. To illustrate this, the bottom panel of Figure 1 plots total hours, employment, and hours per worker during five recession-recovery episodes: 1970:Q1, 1975:Q1, 1982:Q4, 1991:Q1, and 2001:Q4. For reference, the figure displays the first difference of the natural logarithm of GDP as well (top panel). We display employment and hours
Figure 1. U.S. cyclical recoveries. Solid vertical lines indicate the troughs, using the NBER dates. Labor data are measures for the entire economy.

per worker relative to their linear trends. The labor margins positively co-move in some recoveries, such as 1975:Q1 and 1982:Q4, but negatively co-move in other episodes, as in 1991:Q1 and 2001:Q4. In addition, fluctuations in hours per worker are quantitatively important for aggregate hours in several recoveries. For instance, at the 1982:Q4 trough, the difference in employment and total hours relative to trend was over two percentage points, whereas four quarters later the gap shrunk to a difference of about one percentage point (see the bottom row, column three). The closing of the gap was due to hours per worker, which was rising on average over the period. Likewise, in the recovery of 2001:Q4, total hours and hours per worker exhibited a short increase two periods after GDP’s trough, while employment steadily declined over the whole episode. As shown in Appendix A, the time-varying comovement in recession-recovery episodes is robust across different detrending methods and alternative data sources.

In the subsequent sections, we focus on developing a model consistent with these patterns in the data.
3 The Model

To study the dynamics of the labor margins, we build a quantitative general-equilibrium model featuring search and matching frictions. The model shares salient details that many have found useful for capturing features of the data. These include habit formation, costs of adjusting the flow of investment, variable capital utilization, and nominal price and wage rigidities.

The modeling of the labor market follows closely the existing literature, with two important generalizations. First, we consider a general class of preferences that allow us to parametrize the strength of short-run wealth effects on the labor supply. Second, we introduce costly hours adjustment. The model nests the standard framework analyzed in the literature, i.e., without hours adjustment costs and including additively separable household preferences.

We abstract from monetary frictions that would motivate a demand for currency and model a cashless economy following Woodford (2003). Below, variables without a time subscript denote non-stochastic values along the balanced growth path.

Household Preferences

The economy is populated by a representative household with a continuum of members along the unit interval. In equilibrium, some family members are unemployed, while others are employed. As is common in the literature, we assume that family members perfectly insure each other against variation in labor income due to changes in employment status, so that there is no ex post heterogeneity across individuals in the household (see Andolfatto, 1996, and Merz, 1995).

Household’s per-period utility depends on current consumption, $C_t$, lagged consumption $C_{t-1}$ (due to the presence of habit formation), and the disutility of hours supplied by employed members: $H_t \equiv \bar{h}_t \int_0^{L_t} v (h_{jt}) dj$, where $L_t$ denotes the mass of employed workers, $h_{jt}$ denotes hours worked by the employed member $j$, and $v (\cdot)$ is a convex function. The term $\bar{h}_t$ denotes an exogenous shock to the marginal disutility of hours worked, which evolves according to $\log \bar{h}_t = (1 - \rho_h) \log \bar{h} + \rho_h \log \bar{h}_{t-1} + \epsilon_{\bar{h}_t}$ with $\epsilon_{\bar{h}_t} \sim N (0, \sigma_{\bar{h}}^2)$. We calibrate the steady-state value of $\bar{h}$ so that steady-state hours per worker are normalized to one. The representative household maximizes the expected intertemporal utility function

$$W_t \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} \beta_s \left[ u (C_s, C_{s-1}, H_t) \right], \quad (1)$$
where $\beta \in (0, 1)$ is the discount factor and $\tilde{\beta}_t$ is an exogenous shock that evolves according to $\log \tilde{\beta}_t = \rho \log \tilde{\beta}_{t-1} + \varepsilon_{\tilde{\beta}_t}$ with $\varepsilon_{\tilde{\beta}_t} \sim N\left(0, \sigma^2_{\tilde{\beta}}\right)$. As discussed in the next section, we consider two alternative specifications for $u(\cdot)$: additively separable preferences (the benchmark in the literature) and non-separable preferences with parametrized wealth effects.

The consumption basket $C_t$ aggregates differentiated consumption varieties, $C_{\omega t}$, in Dixit-Stiglitz form: $C_t = \int_0^1 C_{\omega t}^{\bar{g}_t/(\bar{g}_t-1)} d\omega$, where $\bar{g}_t > 1$ is the exogenous elasticity of substitution across goods. We assume that $\bar{g}_t$ follows the stochastic process $\log \bar{g}_t = \rho \bar{g}_t \log \bar{g}_{t-1} + (1 - \rho \bar{g}_t) \log \bar{g} + \varepsilon_{\bar{g}_t}$, where $\varepsilon_{\bar{g}_t} \sim N \left(0, \sigma^2_{\bar{g}}\right)$, which, following the literature, we refer to as a price markup shock. The corresponding price index is given by: $P_t = \left[\int_0^1 P_{\omega t}^{1-\bar{g}_t} d\omega\right]^{1/(1-\bar{g}_t)}$, where $P_{\omega t}$ is the price of variety $\omega$.

**Production**

There are two vertically integrated production sectors. In the upstream sector, perfectly competitive firms use capital and labor to produce a homogenous intermediate input. In the downstream sector, monopolistically competitive firms purchase intermediate inputs and produce the differentiated varieties that are sold to consumers. This production structure is common in the search-and-matching literature featuring nominal rigidities and monopolistic competition, as it simplifies the introduction of labor market frictions in the model; see, for instance, Gertler et al. (2008), Ravenna and Walsh (2011), and Trigari (2009).

**Intermediate Input Producers**

There is a unit mass of perfectly competitive intermediate producers. Production requires capital and labor. Within each firm there is a continuum of jobs; each job is executed by one worker. Capital is perfectly mobile across firms and jobs and there is a competitive rental market in capital. All jobs produce with identical exogenous productivity $\bar{A}_t$. We assume that the growth rate of technology, $\bar{g}_{At} \equiv \bar{A}_t/\bar{A}_{t-1}$, follows the stochastic process: $\log \bar{g}_{At} = \rho \bar{g}_A \log \bar{g}_{At-1} + (1 - \rho \bar{g}_A) \log \bar{g}_A + \varepsilon_{\bar{g}_{At}}$, where $\varepsilon_{\bar{g}_{At}} \sim N \left(0, \sigma^2_{\bar{g}_A}\right)$. Since all jobs produce with productivity $\bar{A}_t$, existing matches use the same capital and hours inputs to produce the same amount of output. For this reason, we omit a job-specific index below.

A filled job in the representative firm $j$ produces $(k_{jt})^\alpha \left(\bar{A}_t \bar{h}_{jt}\right)^{1-\alpha}$ units of output, where $k_{jt}$ is the stock of capital allocated to the job and $\bar{h}_{jt} \equiv h_{jt} \left[1 - \frac{\phi_h}{2} \left(\frac{h_{jt} - h_j}{h_j}\right)^2\right]$ denotes hours per worker
net of a cost of adjustment $\phi_h \geq 0$. The latter adjustment cost captures various frictions that constrain the ability of firms to adjust hours per worker—for instance, technological constraints due to set-up costs and coordination issues. Producer’s output exhibits constant returns to scale in total effective hours and capital:

$$Y_{jt}^I \equiv L_{jt} \left( k_{jt} \right)^\alpha \left( \bar{A}_t \bar{h}_{jt} \right) \bar{A}_t \left( L_{jt} \bar{h}_{jt} \right)^{1-\alpha},$$

where $L_{jt}$ is the measure of jobs within the firm and $K_{jt} \equiv L_{jt}k_{jt}$.

The relationship between a firm and a worker can be severed for exogenous reasons. We denote by $\lambda$ the fraction of jobs that are exogenously destroyed in each period. Job creation is subject to matching frictions. To hire a new worker, firms have to post a vacancy, incurring a real cost $\bar{A}_t \kappa_{jt}$, where $\kappa_{jt} \equiv \kappa V_{jt}^{1+\tau} / (1 + \tau)$. This specification implies that total vacancy costs are convex in the number of posted vacancies, $V_{jt}$, an assumption that is consistent with the evidence in Merz and Yashiv (2007). We let the vacancy cost drift with the level of technology to ensure balanced growth. The probability of finding a worker depends on a constant returns to scale matching technology, which converts aggregate unemployed workers $U_t$ and aggregate vacancies $V_t$ into aggregate matches $M_t = \chi U_t V_t^{1-\varepsilon}$, where $0 < \varepsilon < 1$. Each firm meets unemployed workers at a rate $q_t \equiv M_t / V_t$.

Finally, as is common practice in the literature, all separated workers are assumed to reenter the unemployment pool; i.e., we abstract from workers’ labor-force participation decisions.

The timing of events, summarized by Figure 2, proceeds as follows. The firm $j$ begins a period with a stock of $L_{jt-1}$ workers, which is immediately reduced by exogenous separations. Aggregate shocks are then realized, and the representative firm posts vacancies $V_{jt}$ and selects the capital stock $K_{jt}$. The timing of the capital decision is immaterial for the equilibrium allocation, since capital can be costlessly adjusted within each firm in a given period (Cahuc et al., 2008). Once the hiring round has been completed, wages and hours per worker are determined, and production

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8 Results are robust to alternative specifications of the costly hours adjustment, including adjustment costs relative to the lagged level of hours and adjustment costs modeled as a resource cost. In addition, results are robust to alternatively modeling habits in hours in the household’s utility function. See Appendix H for details and discussion about the fit of these alternative specifications.

9 Hall (2005) and Shimer (2005) argue that, in the U.S. data, the separation rate varies little over the business cycle, although part of the literature disputes this position; see Davis et al. (1998) and Fujita and Ramey (2009). Our estimates are robust to considering exogenous fluctuations in the separation margin together with a measure of the separation rate included in the observables. See Appendix H for more details.

10 See Campolmi and Gnocchi (2016) for a New Keynesian model with search and matching frictions that incorporates a participation decision.
Timing of Events in the Labor Market

Exogenous job destruction

Realization of aggregate shocks

Search and Matching:
producers post vacancies;
unemployed workers search for jobs; \( m_t \) new matches are formed

Hours-per match determination

Wage bargaining

Production and Market Clearing

Beginning of Period \( t \)

End of Period \( t \)

Figure 2. Timing of the events in the labor market.

occurs. The law of motion of employment is given by:

\[
L_{jt} = (1 - \lambda)L_{jt-1} + q_t V_{jt}.
\]

Thus, labor-market matching occurs within a period, which, as discussed by Arseneau and Chugh (2012), is empirically descriptive of U.S. labor-market flows at quarterly frequencies.

Following Arseneau and Chugh (2008), we use Rotemberg’s (1982) model of a nominal rigidity and assume that firms face a quadratic cost of adjusting the hourly nominal wage rate, \( w_{jt}^n \). We opt for this formulation rather than staggered wage adjustment (as for instance in Gertler et al., 2008), since it ensures the existence of a representative producer in the presence of endogenous hours adjustment and parametrized wealth effects, providing tractability.\(^{11}\)

The real, per-worker cost of changing the nominal wage between period \( t - 1 \) and \( t \) is

\[
\Gamma_{w,t} \equiv \frac{\phi_t A_t}{2} \left( \frac{w_{jt}^n}{w_{jt-1}^n} \pi_{Ct-1}^{\tau_{-1}^{w}} \pi_{Ct}^{\tau_{w}} - \bar{g}_t A_t \right)^2,
\]

\(^{11}\)Even in a model abstracting from the intensive margin, wage bargaining with Rotemberg or Calvo wage rigidities are not observational equivalent. As discussed by Gertler and Trigari (2009), the wage dispersion implied by staggered Nash bargaining generates a spillover effect on the average wage that is absent with convex wage adjustment costs. However, from a quantitative perspective, these alternative specifications have second-order effects, since a version of our model that abstracts from fluctuations in hours per worker yields aggregate dynamics that are virtually identical to Gertler et al. (2008)—results are available upon request.
where $\phi^w \geq 0$ is in units of consumption, $\pi_{Ct} \equiv P_t/P_{t-1}$ is the gross CPI inflation rate, and $\iota_w \in [0,1]$ measures the degree to which nominal wage adjustment is indexed to previous price inflation. If $\phi^w = 0$, there is no cost of wage adjustment. Similar to the vacancy cost, the wage adjustment cost is tied to the level of technology $\tilde{A}_t$ to ensure balanced growth.

Intermediate input producers sell their output to final producers at a real price $\varphi_t$ in units of consumption. The present discounted value of the stream of profits is given by:

$$\Pi^{I}_{jt} \equiv E_t \left\{ \sum_{t=s}^{\infty} \beta_{t,s+1} \left[ \varphi_{jt} Y_{jt} - \frac{w^n_{jt} h_{jt}}{P_t} L_{js} - \Gamma w_{js} L_{js} - r K_s K_{js} - \kappa \tilde{A}_s \frac{V^{1+\tau}}{1+\tau} \right] \right\},$$

where $\beta_{t,t+1} \equiv \beta u_{Ct+1}/u_{Ct}$ is the household stochastic discount factor (the term $u_{Ct}$ denotes the marginal utility of consumption). The representative producer chooses $V_{jt}$, $L_{jt}$, and $K_{jt}$ to maximize (4) subject to (2) and (3). When making these decisions, the firm anticipates that both the hourly wage $w_{jt}$ and hours per worker $h_{jt}$ do not depend on the scale of the firm, so that $\partial w^n_{jt}/\partial L_{jt} = \partial w^n_{jt}/\partial K_{jt} = \partial h_{jt}/\partial L_{jt} = \partial h_{jt}/\partial K_{jt} = 0$. These well-known results, proved below, obtain under the standard assumptions of perfectly competitive intermediate producers, constant returns to scale, and capital rented in competitive markets (coupled with individual Nash wage bargaining and neoclassical determination of hours per worker).

The first-order condition for $K_{jt}$ equates the marginal revenue product of capital to its rental cost: $\varphi_t \beta \left( \frac{K_{jt}}{A_t h_{jt} L_{jt}} \right)^{\alpha-1} = r K_t$, implying that the capital-total hours ratio is symmetric across producers, since it only depends on aggregate variables. Let $S^{I}_{jt}$ denote the Lagrange multiplier on the constraints (3), representing the value to the firm of hiring an extra worker. The first-order condition for $L_{jt}$ implies:

$$S^{I}_{jt} = (1 - \alpha) \varphi_t \left( \frac{K_{jt}}{A_t h_{jt} L_{jt}} \right)^{\alpha} \tilde{A}_t h_{jt} - \frac{w^n_{jt} h_{jt}}{P_t} - \Gamma w_{jt} + E_t \beta_{t,t+1} (1 - \lambda) S^{I}_{jt+1}.$$  

Intuitively, the value of a job to the firm corresponds to the expected, present discounted value of the streams of profits from the match—the difference between the value of the marginal product and the wage payment to the worker minus the cost of adjusting the nominal wage. Using the first-order condition for capital, the firm’s surplus can be written as:

$$S^{I}_{jt} = (1 - \alpha) \varphi_t \left( \frac{r K_t}{\varphi_t \alpha} \right)^{\alpha-1} \tilde{A}_t h_{jt} - \frac{w^n_{jt} h_{jt}}{P_t} - \phi^w \tilde{A}_t \left( \frac{w^n_{jt} \varphi_t}{w^n_{jt-1} \pi_{Ct-1} \pi_{Ct} - \tilde{g}_A} \right)^2 + E_t \beta_{t,t+1} (1 - \lambda) S^{I}_{jt+1}.$$
Finally, the first-order condition for vacancies equates the cost of filling a vacancy to the value of a filled position:

$$\kappa \bar{A}_t \frac{V_{jt}}{q_t} = S_{jt}^f. \tag{6}$$

Equations (5) and (6) imply a standard job creation condition:

$$\kappa \bar{A}_t \frac{V_{jt}}{q_t} = (1 - \alpha) \left(1 - \frac{w_{jt}^n h_{jt}}{P_t} - \Gamma_{w_{jt}} + \kappa (1 - \lambda) E_t \beta_{t,t+1} \frac{\bar{A}_{t+1} V_{jt+1}^f}{q_{t+1}} \right).$$

Forward looking iteration of the job creation equation implies that, at the optimum, the expected, present discounted value of the stream of profits generated by a match over its expected lifetime is equal to the cost of filling a vacancy, \(\kappa \bar{A}_t V_{jt}/q_t\).

**Hours Determination**

As is common practice in the literature, we assume that workers and firms cooperatively choose hours per worker to maximize their joint surplus. As shown in Appendix B, this implies that \(h_{jt}\) adjusts to the point where the worker’s marginal rate of substitution between consumption and leisure is equal to the value of the marginal product of an extra hour worked:

$$\frac{\partial u_{h_{jt}}}{\partial h_{jt}} = u_C \left(1 - \alpha \right) (1 - \alpha) \varphi_t \left(\frac{w_{jt} h_{jt}}{P_t} \right)^{\frac{\alpha}{\alpha - 1}} A_t \Delta_{h_{jt}},$$

where \(u_{h_{jt}}\) denotes the worker’s disutility from supplying hours (i.e., \(\partial u_{h_{jt}}/\partial h_{jt}\) is the worker’s marginal disutility from supplying an extra hour) and

$$\Delta_{h_{jt}} = \frac{\partial \tilde{h}_{jt}}{\partial h_{jt}} = \frac{\tilde{h}_{jt}}{h_{jt}} - \phi_h h_{jt} \left(\frac{h_{jt}}{h_j} \right).$$

Notice that, while up to a first-order approximation \(\tilde{h}_{jt} = h_{jt}\), the hours adjustment cost \(\phi_h\) affects aggregate dynamics through the term \(\Delta_{h_{jt}}\). Using the first-order condition for capital, the optimal choice of hours per worker implies:

$$\frac{\partial u_{h_{jt}}}{\partial h_{jt}} = u_C \left(1 - \alpha \right) (1 - \alpha) \varphi_t \left(\frac{w_{jt} h_{jt}}{P_t} \right)^{\frac{\alpha}{\alpha - 1}} A_t \Delta_{h_{jt}}. \tag{7}$$

Equation 7 shows that \(h_{jt}\) only depends on aggregate conditions, i.e., \(h_{jt} = h_t\) is invariant to the scale of the firm. Finally, \(h_{jt}\) does not directly depend on the hourly wage \(w_{jt}\).
Wage Bargaining

The nominal wage is the solution to an individual Nash bargaining problem, and the wage payment divides the match surplus between workers and firms. Due to the presence of nominal rigidities, we assume that bargaining occurs over the nominal wage rather than the real wage, as in Arseneau and Chugh (2008), Gertler et al. (2008), and Thomas (2008). With zero costs of nominal wage adjustment ($\phi^w = 0$), the real wage would be identical to the one obtained from bargaining directly over the real wage. This is no longer the case in the presence of wage adjustment costs. Moreover, both parties account for the fact that $\frac{\partial h_t}{\partial w_{jt}} = 0$, as shown above.

Let $\bar{\eta}_t \in (0, 1)$ be the weight given to the worker’s individual surplus in Nash bargaining. We assume that $\bar{\eta}_t$ follows the process: $\log \bar{\eta}_t = \rho \bar{\eta}_{t-1} + \varepsilon_{\bar{\eta}t}$, where $\varepsilon_{\bar{\eta}t} \sim N(0, \sigma^2_{\bar{\eta}})$. Exogenous fluctuations in the worker’s bargaining power are the counterpart of wage-markup shocks typically assumed in the estimation of New Keynesian models that abstract from search-and-matching frictions. The firm and the worker maximize the Nash product $\left(S^f_{jt}\right)^{1-\bar{\eta}_t} \left(S^w_{jt}\right)^{\bar{\eta}_t}$, where $S^f_{jt}$ is defined as in (5) and $S^w_{jt}$ denotes the worker’s surplus:

$$S^w_{jt} = \frac{w^m_{jt}}{P_t} h_t - b\tilde{A}_t - \frac{u_{ht}}{u_{Ct}} + E_t \left[\beta_{t+1} (1 - \lambda) S^w_{jt+1} \left(1 - \frac{M_{t+1}}{U_{t+1}}\right)\right],$$

where we have used the fact that $h_{jt} = h_t$, as shown above. The worker’s surplus corresponds to the expected present discounted value of wage payments over the lifetime of the match minus the expected present discounted value of the flow value of unemployment, including unemployment benefits from the government $b\tilde{A}_t$ (financed with lump sum taxes), and the utility gain from leisure in terms of consumption, $u_{ht}/u_{Ct}$.

The first-order condition with respect to $w^m_{jt}$ implies the following sharing rule: $\eta_{w,jt} S^f_{jt} = (1 - \eta_{w,jt}) S^w_{jt}$, where $\eta_{w,jt}$ is the effective bargaining share of workers:

$$\eta_{w,jt} = \frac{\bar{\eta}_t h_t}{\bar{\eta}_t h_t - (1 - \bar{\eta}_t) \left(\partial S^f_{jt}/\partial w^m_{jt}\right)}.$$

(See Appendix C for the expression of $\partial S^f_{jt}/\partial w^m_{jt}$.) As in Gertler and Trigari (2009), the effective

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12Given the assumption of complete markets within the households, we follow the existing literature and formulate the bargaining problem as though the worker is interested in maximizing the expected discounted income. As discussed in Rogerson et al. (2005), when markets are complete, this approach is equivalent to maximizing expected utility even if the worker is risk averse, since s(he) can maximize utility by first maximizing income and then smoothing consumption.
bargaining share is time-varying due to the presence of wage adjustment costs. Absent these costs, the bargaining share would be exogenous, $\eta_{w,t} = \bar{\eta}_t$. Importantly, the wage rigidity implies that $\eta_{w,t}$ is countercyclical, amplifying employment fluctuations in response to aggregate shocks, as first noted by Gertler and Trigari (2009).

It is straightforward to verify that $w_{jt}^n$ does not depend on the scale of the firm, i.e., $\partial w_{jt}^n / \partial L_{jt} = \partial w_{jt}^n / \partial K_{jt} = 0$. To see this, substitute equation (7) into the definition of the worker’s and firm’s surplus, $S_{jt}^w$ and $S_{jt}^f$, and use the first-order condition for capital to eliminate the capital-labor ratio in $S_{jt}^f$. Then, since all the intermediate firms produce with identical technology $\bar{A}_t$, there is a symmetric equilibrium in which $K_{jt} = K_t$, $L_{jt} = L_t$, $h_{jt} = h_t$, $V_{jt} = V_t$, and $w_{jt}^n = w_t^n$.

In turn, nominal hourly wage inflation, defined by $\pi_{wt} \equiv w_{t-1}^n / w_t^n - 1$, is linked to CPI inflation by $\pi_{wt} = (w_t / w_{t-1}) \pi_{Ct}$, where $w_t \equiv w_t^n / P_t$ denotes the real hourly wage. Finally, searching workers in period $t$ are equal to the mass of unemployed workers: $U_t = 1 - (1 - \lambda) L_{t-1}$.

**Final Goods Production**

A continuum of monopolistically competitive final-sector firms produce differentiated varieties using the intermediate input. The producer $\omega$ faces the following demand: $Y^C_{\omega t} = (P_{\omega t} / P_t)^{-\bar{\theta}} Y_t^C$, where $Y_t^C$ denotes aggregate demand of the final consumption basket.

We introduce price-setting frictions by following Rotemberg (1982) and assume that final producers must pay a quadratic price adjustment cost. We also allow for price indexation by assuming that final producers index price changes to past CPI inflation, so that price adjustment costs take the form:

$$\frac{\phi^p}{2} \left( \frac{P_{\omega t}}{P_{\omega t-1}} \pi_{Ct-1}^{-\bar{\theta} - 1} - 1 \right)^2 P_{\omega t} Y^C_{\omega t},$$

where $\phi^p \geq 0$ determines the size of the adjustment cost (prices are flexible if $\phi^p = 0$) and $\bar{\theta} \in [0, 1]$ is the indexation parameter.

Optimal price setting implies that the (real) output price $P_{\omega t} / P_t$ is equal to a markup over the real marginal cost $\varphi_t$: $P_{\omega t} / P_t = \left( \bar{\theta}_t / (\bar{\theta}_t - 1) \right) \Xi_{\omega t} \varphi_t$, where

$$\Xi_{\omega t} \equiv 1 - \frac{\phi^p}{2} \left( \pi_{\omega t} \pi_{Ct-1}^{-\bar{\theta}} \pi_C^{-\bar{\theta} - 1} - 1 \right)^2 + \frac{\phi^p}{\bar{\theta}_t - 1} \left\{ \pi_{Ct-1} \pi_{\omega t}^{-\bar{\theta} - 1} \left( \pi_{\omega t} \pi_{Ct-1}^{-\bar{\theta} - 1} \pi_C^{-\bar{\theta} - 1} - 1 \right) \right. - E_t \left( \beta_{t+1} \pi_{Ct+1} \pi_{Ct-1}^{-\bar{\theta} - 1} \pi_C^{-\bar{\theta} - 1} - 1 \right) \pi_{Ct+1} \pi_{\omega t}^{-\bar{\theta} - 1} \pi_{Ct} \pi_C^{-\bar{\theta} - 1} Y^C_{\omega t} \right\},$$

and $\pi_{\omega t} \equiv P_{\omega t} / P_{\omega t-1}$. There are two sources of endogenous markup variation in the model. First, the cost of adjusting prices gives firms an incentive to change markups over time in order to smooth
price changes across periods. Second, exogenous shocks to the firms’ market power result in time-varying markups even in the absence of price stickiness. In the symmetric equilibrium, $P_{\omega t} = P_t$ and $\Xi_{\omega t} = \Xi_t$. As a consequence, $\pi_{\omega t} = \pi_t = \pi_C t$.

**Household Budget Constraint and Optimal Intertemporal Decisions**

The household enters period $t$ with nominal private bond holdings $B_t$, earning a gross interest rate $i_t$. The household also accumulates physical capital and rents it to intermediate input producers in a competitive capital market. Investment in the physical capital stock, $I_{Kt}$, requires the use of the same composite of all available varieties as the basket $C_t$. We introduce convex adjustment costs in physical investment and variable capital utilization. The utilization rate of capital is set by the household. Thus, effective capital rented to firms, $K_t$, is the product of physical capital, $\tilde{K}_t$, and the utilization rate, $u_{Kt}$: $K_t = u_{Kt} \tilde{K}_t$. Increases in the utilization rate are costly because higher utilization rates imply faster capital depreciation. We assume a standard convex depreciation function: $\delta_{Kt} = \delta_{K0} + \delta_{K1} (u_{Kt} - u_K) + \delta_{K2} (u_{Kt} - u_K)^2$. Physical capital, $\tilde{K}_t$, obeys a standard law of motion:

$$\tilde{K}_{t+1} = (1 - \delta_{Kt}) \tilde{K}_t + \bar{P}_{Kt} \left[ 1 - \frac{\nu_K}{2} \left( \frac{I_{Kt}}{I_{Kt-1}} - \bar{g}_A \right)^2 \right] I_{Kt}, \quad (9)$$

where $\nu_K > 0$ is a scale parameter, and $\bar{P}_{Kt}$ is an investment specific shock. The latter is a source of exogenous variation in the efficiency with which the final good can be transformed into physical capital, and thus into tomorrow’s capital input. Justiniano et al. (2010) suggests that this variation might stem from technological factors specific to the production of investment goods and also from disturbances to the process by which these investment goods are turned into productive capital. The investment shock evolves via the process $\log \bar{P}_{Kt} = \rho \bar{P}_{Kt-1} + \varepsilon_{\bar{P}_{Kt}}$, where $\varepsilon_{\bar{P}_{Kt}} \sim iid N \left( 0, \sigma^2_{P_K} \right)$.

The per-period household’s budget constraint is:

$$P_tC_t + P_tI_{Kt} + B_{t+1} = i_t B_t + w_t^n h_t L_t + r_{Kt} P_t K_t + b \bar{A}_t (1 - L_t) P_t + P_t \Pi'_t + P_t \int_0^1 \Pi'_t F(i) di + T^q_t, \quad (10)$$

where $T^q_t$ is a nominal lump-sum tax from the government.

The household maximizes its expected intertemporal utility subject to (9) and (10). The Euler equation for capital accumulation requires: $\zeta_{Kt} = E_t \left\{ \beta_{t,t+1} \left[ r_{t+1} u_{Kt+1} + (1 - \delta_{Kt+1}) \zeta_{Kt+1} \right] \right\}$, where $\zeta_{Kt}$ denotes the shadow value of capital (in units of consumption), defined by the first-order
condition for investment $I_{Kt}$:

$$\zeta_{Kt}^1 = \left[1 - \frac{\nu_K}{2}\Gamma_{Kt}^2 \Gamma_{Kt} (\Gamma_{Kt} + 1) + \nu_K \beta_{t+1}\mathbb{E}_t \left[ \frac{\zeta_{Kt+1}}{\zeta_{Kt}} (\Gamma_{Kt+1} + 1)^2 \right] \right]^{-1},$$

where $\Gamma_{Kt} \equiv (I_{Kt}/I_{Kt-1}) - 1$. The optimal condition for capital utilization implies: $r_{Kt} = \zeta_{Kt}[\delta_{K1} + \delta_{K2}(u_{Kt-1})]$. Finally, the Euler equation for bond holdings implies: $1 = i_tE_t[\beta_{t+1}/(1 + \pi_{Ct+1})]$.

**The Government and Market Clearing**

Fiscal policy is Ricardian, since the government finances its budget deficit with lump-sum taxes each period. Public spending is determined exogenously, $G_t = \bar{g}_t$, where the exogenous government spending shock $\bar{g}_t$ follows the process $\log \bar{g}_t = \rho_{\bar{g}} \log \bar{g}_t + (1 - \rho_{\bar{g}}) \log \bar{g} + \varepsilon_{\bar{g}t}$, with $\varepsilon_{\bar{g}t} \sim iid N(0, \sigma_{\bar{g}}^2)$.

The monetary authority sets the nominal interest rate following a feedback rule of the form

$$\frac{i_t}{\bar{i}_t} = \left(\frac{i_{t-1}}{\bar{i}_t}\right)^{\theta_{it}} \left[\left(\frac{\pi_{Ct}}{\pi_C}\right)^{\theta_{\piC}} \left(\frac{Y_{gt}}{\bar{Y}}\right)^{\theta_{Yg}}\right]^{1-\theta_{it}} \left(\frac{Y_{gt}}{\bar{Y}_{gt-1}}\right)^{\theta_{Yg}}\bar{i}_{it},$$

where $\bar{i}$ is the steady-state gross nominal interest rate. The interest rate responds to deviations of inflation and the GDP gap, $Y_{gt}$, from their long-run values, as well as to the growth rate of the GDP gap, $Y_{gt}/Y_{gt-1}$. GDP is defined as $Y_t \equiv C_t + I_{Kt} + G_t$. Consistent with Woodford (2003), we define the GDP gap as the deviation of GDP from its level prevailing under flexible prices and wages and absent inefficient shocks (i.e., absent markup and bargaining power shocks). The monetary policy rule is subject to a shock, $\bar{i}_{it}$, which evolves according to $\log \bar{i}_{it} = \rho_{\bar{i}} \log \bar{i}_{it-1} + \varepsilon_{\bar{i}t}$, with $\varepsilon_{\bar{i}t} \sim iid N(0, \sigma_{\bar{i}}^2)$.

In the symmetric equilibrium, bonds are in zero net supply: $B_t = B_{t+1} = 0$. Thus, combining the household’s and government’s budget constraints yields the following aggregate resource constraint:

$$Y_t^C = \left[1 - \frac{\nu}{2} \left(\pi_{Ct}\pi_C^{-1} - \pi_{Ct-1}^{-1}\right)^2 \right] = \left[C_t + I_{Kt} + \kappa_t \bar{A}_t V_t + G_t + \frac{\phi^w \bar{A}_t}{2} \left(\pi_{Ct}\pi_C^{-1} - \pi_{Ct-1}^{-1} - \bar{g}_A\right)^2 L_t.\right]$$

Total output produced by firms must equal the sum of market consumption, investment in physical capital, the costs of posting vacancies, the purchase of goods from the government, and the cost of changing prices and wages. Labor market clearing implies $Y_t^C = Y_t'$.

The model contains 15 equations that determine 15 endogenous variables: $i_t$, $\pi_{Ct}$, $\pi_{wt}$, $C_t$, $L_t$, $V_t$, $M_t$, $h_t$, $w_t$, $\varphi_t$, $\bar{K}_{t+1}$, $I_{Kt}$, $\zeta_{Kt}$, $u_{Kt}$, $r_{Kt}$, and 14 definitions ($U_t$, $S_t^f$, $S_t^w$, $h_t$, $q_t$, $u_{Ct}$, $u_{ht}$, $\bar{h}_t$, $\Delta_{ht}$, $\Delta_{ht}$).
δKt, κt, ηwt, Ξt, and Ygt). As detailed below, the terms ucit, and uth dependent on the specification of the household’s utility, u(·). Additionally, the model features 8 exogenous disturbances: gAt, 3βt, 3ht, 3it, 3Pkt, 3ıt, and 3gt. Consumption, investment, capital, the real wage, and GDP, (together with YCt, Stf, Ssw, and ucit) fluctuate around a stochastic balanced growth path, since the level of technology has a unit root. We rewrite the model in terms of detrended variables and compute the log-linear approximation around the non-stochastic steady state. The details of these steps can be found in Appendix D, along with the full set of stationarized equilibrium conditions (and their log-linear approximations). We then solve the resulting linear system of rational-expectation equations to obtain the transition equations, which are linked to data with an observation equation to form the state-space model used for estimation.

4 Baseline Model: Separable Preferences and Frictionless Hours Adjustment

In this section, we study a model that corresponds to the baseline version considered in the literature: separable preferences and frictionless hours-per-worker adjustment—see for instance, Andolfatto (1996), Arseneau and Chugh (2008), Christiano et al. (2011), Merz (1995), Ravenna and Walsh (2012), and Trigari (2009). Using a Bayesian prior-predictive analysis, we show a priori the baseline model can account for the relative variability or comovement of the labor margins. However, conditioning the model on macro and labor observables results in posterior estimates that imply counterfactual labor-market dynamics.

Preference Specification

We set φh = 0 and assume that

\[ W_t \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} \beta_s \left[ \log(C_t - hC_{t-1}) - \bar{h}_t \int_0^{L_t} \frac{h_{j}^{1+\omega}}{1+\omega} dj \right], \]

where hC is the degree of habit formation. Consumption utility is logarithmic to ensure the existence of a balanced growth path in the presence of non-stationary technological progress. This preference specification implies that the disutility of labor supply is uhj,t \equiv 3βt3ht3h_{jt}^{1+\omega}/(1+\omega), while the marginal disutility of working an extra hour is ∂uhj,t/∂h_{jt} \equiv 3βt3ht3h_{jt}^{\omega}. Table A.5 summarizes the equilibrium conditions of the baseline model.

We estimate the model with U.S. quarterly data from 1965:Q1 to 2007:Q4. Details of the estimation procedure, as well as the data construction and linkages to observables, are presented.
in Appendix E. For the baseline estimation, we end the estimation prior to the recent zero lower bound episode.\textsuperscript{13} Our initial estimation includes seven observables commonly employed in the literature (e.g., Christiano et al., 2005, and Smets and Wouters, 2007): the log difference of real aggregate consumption, investment, GDP, and real wages, the log difference of the GDP deflator, the Federal Funds rate, and the log of economy-wide total hours worked. To avoid stochastic singularity, we include seven structural shocks. To facilitate comparison with the literature (i.e., Christiano et al., 2011, and Gertler et al., 2008), our first specification assumes that shocks to the exogenous component of the worker’s bargaining power, $\bar{\eta}_t$, are the only disturbance directly affecting the labor market, i.e., $\bar{h}_t = \bar{h}$ for any $t$.\textsuperscript{14}

In addition, we estimate a second version of the model that includes the hours supply shock, $\bar{h}_t$, and one ancillary observable, the log of economy-wide employment.\textsuperscript{15} In this case, we consider the linearly detrended employment series from Section 2. Using information on both margins of labor adjustment helps identify key labor parameters such as the Frisch elasticity. Moreover, the inclusion of the hours supply shock gives the model a better chance to match the dynamics of the labor margins.

**Prior Distributions**

We impose dogmatic priors for some parameters. The household discount factor $\beta$ is set to 0.99, the capital share in production $\alpha$ is 0.3, and depreciation rate $\delta_K = \delta_{K0}$ is 0.025. The parameter $\delta_{K1}$ is calibrated so that the utilization rate is one in steady state ($u_K = 1$). For future reference, we define the parameter $\zeta$ such that $\delta_{K2}/\delta_{K1} = \zeta/(1 - \zeta)$. The steady-state price markup is set at 1.1.\textsuperscript{16} Steady-state government spending is fixed at 20 percent of GDP, which equals the post-war average for all levels of government spending. Following standard practice in the literature, we use independent evidence for the average quarterly separation rate $\lambda$ and the elasticity of matches to unemployment, $\varepsilon$. In particular, we choose $\lambda = 0.105$ based on the observation that jobs last on average about two and half years in the U.S. economy (Shimer, 2005). We set $\varepsilon$ to be equal to 0.5, the midpoint of the evidence typically cited in the literature and within the range of plausible values (0.5 to 0.7) reported by Petrongolo and Pissarides (2006). Finally, we set the cost of posting

\textsuperscript{13}See Hirose and Inoue (2015) for a discussion of how the ZLB can bias estimates of log-linearized model parameters.
\textsuperscript{14}In appendix H, we show that our results are not sensitive to the alternative possibility of focusing on stochastic fluctuations in the disutility of hours worked, $\bar{h}_t$, while keeping constant the worker’s bargaining power, i.e., $\bar{\eta}_t = 1$.
\textsuperscript{15}Using any two of the three labor market series (total hours, employment, or hours per worker) for estimation is observationally equivalent, since we abstract from measurement error.
\textsuperscript{16}As shown in Appendix H, our results are robust to estimating the steady-state price markup.
a vacancy, \( \kappa \), and the matching efficiency parameter, \( \chi \), to match the quarterly average job finding probability, \( M/U \), and the average probability of filling a vacancy, \( q \). For the U.S., the former is equal to 0.95, while the latter is 0.9 (Shimer, 2005).

Table A.2 in the Appendix lists the prior distributions for the remaining parameters. Our priors for common parameters are similar to those in Smets and Wouters (2007). We set the price stickiness parameter, \( \phi^p \), to a value that would replicate the frequency of price adjustment in a Calvo-type Phillips curve in the absence of strategic price complementarities. For comparability with the literature, we directly estimate the related Calvo parameter \( \xi^p \).\(^{17}\) In contrast, for the wage stickiness parameter, no direct mapping between Calvo and Rotemberg pricing exists, even in a linearized setup. Thus, we employ a prior for \( \phi^w \) that permits a broad degree of stickiness.

The estimated labor market parameters include the steady-state value of the workers’ bargaining power \( \bar{\eta} \), the replacement rate \( b/wh \), and the degree of convexity in the cost of posting vacancies \( \tau \). The first two have priors similar to those in Gertler et al. (2008). Finally, the bargaining power, price markup, and investment shocks are normalized to enter with a unitary coefficient in the log-linearized equations that determine wages, inflation, and investment, respectively. The priors for the standard deviations of shocks are chosen to generate similar volatilities between the variables they directly impact and their data counterparts, as is common practice in the literature.

**Prior-Predictive Analysis**

Prior to confronting the model with data, we first employ a prior-predictive analysis in the spirit of Geweke (2010, Chapter 3) to identify the entire range of sample moments of labor and macro variables admitted by the model structure. This analysis lets us determine whether the model inherently can account for the labor margins. That is, whether reproducing the positive comovement between the labor margins as well as matching their relative volatility is possible with certain parameterizations and shocks, before imposing the discipline of the data. In other contexts, this approach has been adopted recently in the literature, e.g. Leeper et al. (2017). The prior-predictive procedure consists of taking draws from our priors, solving the model, and calculating model sample statistics. We then repeat the procedure for a large set of draws to account for the range of parameterizations implied by our priors.\(^{18}\)

\(^{17}\)\( \xi^p \) is related to \( \phi^p \) via the mapping \( \phi^p = \left[ (\bar{\theta} - 1) / \bar{\theta} \right] \xi^p / (1 - \xi^p)(1 - \xi^p \beta) \).

\(^{18}\)We sample 10,000 draws from our priors. For each draw, we simulate 100 samples from the model with the same length as our dataset for estimation, after first discarding the first 100 observations. We compute statistics for each of these samples.
Table 2 displays the 90-percent prior probability bands of the shares of the variance of total hours attributed to hours per worker, employment, and their covariance for the baseline model, where the bands reflect the individual distribution of each variance share. We calculate these statistics for the two versions of the model we estimate, i.e. with seven or eight structural shocks. A priori, the baseline model admits a broad set of variance shares, whose range includes the data counterparts. As discussed in the next subsection, the comovement between the intensive and extensive margins depends on the direction and strength of the wealth effect on labor supply. For the relative volatilities of the two labor margins, what matters is the relative cost of adjusting hours and employment.

Although the model can individually replicate the variance shares in the data, parameters that correctly match one statistic often do so at the expense of others. For instance, for the model with eight structural shocks, 24% of the parameter draws that produce the positive comovement between the intensive and extensive margins \((\beta_{cov} > 0)\) imply a counterfactually high variance of hours per worker relative to the variance of employment. In contrast, only 5% of the parameter draws for which \(\beta_{cov} < 0\) imply this counterfactual relationship. In addition, parameters that produce empirically plausible variance shares of the labor margins, can imply counterfactual relationships between labor and macro variables and macro variables with each other. To illustrate this point, we used the Kalman filter with our observables for estimation to compute the log-likelihood value from each prior parameter draw. A higher likelihood value implies greater model fit to the data. For the parameter draws that produce the positive \(\beta_{cov}\), the 90-percentile bands for the log-likelihood values range from -38,310 to -11,058. In contrast, the 90-percentile bands from the parameters draws that do not produce the positive \(\beta_{cov}\) have much larger values, ranging from -33,627 to -3,504. In other words, parameterizations that account for labor market outcomes still may imply counterfactual implications for other aggregate series, leading to a worse fit. As a result, assessing the ability of the model to account for the cyclical properties of the labor margins requires structural identification of the model parameters, which we address next.
Table 2: Prior-Predictive Analysis and Posterior Estimates

<table>
<thead>
<tr>
<th></th>
<th>$\beta_h$ (var($h_t$))</th>
<th>$\beta_L$ (var($L_t$))</th>
<th>$\beta_{cov}$ (2cov($h_t$, $L_t$))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>0.18</td>
<td>0.51</td>
<td>0.31</td>
</tr>
<tr>
<td><strong>Prior Distribution</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Model 7 shocks</td>
<td>[0.007, 0.39]</td>
<td>[0.19, 1.67]</td>
<td>[-0.82, 0.47]</td>
</tr>
<tr>
<td>Baseline Model 8 shocks</td>
<td>[0.009, 0.42]</td>
<td>[0.20, 1.68]</td>
<td>[-0.85, 0.45]</td>
</tr>
<tr>
<td><strong>Posterior Distribution</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline Model 7 shocks</td>
<td>[0.03, 0.22]</td>
<td>[0.64, 1.21]</td>
<td>[-0.35, 0.25]</td>
</tr>
<tr>
<td>Baseline Model 8 shocks</td>
<td>[0.16, 0.76]</td>
<td>[0.22, 0.89]</td>
<td>[-0.43, 0.37]</td>
</tr>
<tr>
<td>Preferred Model</td>
<td>[0.13, 0.56]</td>
<td>[0.20, 0.71]</td>
<td>[-0.05, 0.44]</td>
</tr>
</tbody>
</table>

Posterior Estimates and Model Fit

To understand how well the baseline model fits the data, we compare a set of statistics implied by the model to their data counterparts. We relegate to the Appendix the posterior estimates of the baseline model and a discussion of these estimates relative to the literature (see Appendix E and Table A.2). Figure 3 plots the correlogram for several aggregate macroeconomic and labor market variables in the data (solid lines), as well as the 90-percent posterior intervals implied by both parameter and small sample uncertainty from the seven observable case (dotted-dashed lines) and the eight observable case (dashed lines). We discuss the results of each case in turn.

Figure 3 compares statistics from the data (solid lines) and the model estimated with seven shocks (dotted-dashed lines). The model cannot account for the correlations between labor market variables and aggregate macroeconomic series. First, the baseline model estimated with seven shocks does not capture the positive correlation between hours and employment nor the relative contributions of the labor margins to the variance of total hours. In particular, the model assigns an almost exclusive role to employment, as the 90 percent posterior bands for the share of the extensive margin to the variance of total hours ($\beta_L$) are between 0.64 and 1.21 (see Table 2), while the data counterpart is only 0.51. The $\beta_{cov}$ ranges from $-0.35$ to 0.25, well short of the positive comovement (0.31) between hours and employment observed in the data. Moreover, the model overstates the correlation between the growth rate of output with total hours or employment at various leads and lags. Even though it correctly reproduces the correlogram between total hours and consumption growth, it does so with a counterfactual comovement of the individual margins with respect to consumption.

As in the prior-predictive analysis, we sample 10,000 draws from the posterior. For each parameter draw, we generate 100 samples of the observable variables from the model with the same length as our dataset, after first discarding 100 initial observations. We compute statistics for each of these samples.
Figure 3. Correlograms from the data (solid lines) and 90 percent posterior intervals from 1) the baseline model with seven shocks (dotted lines) and 2) the baseline model with eight shocks (dashed lines).
Prima facie, the poor performance of the model with seven shocks could reflect that the model is estimated with only one labor market observable. However, simply adding information about the labor market by increasing the set of observables (and shocks) to include simultaneously employment (or hours per worker) and total hours does not improve the performance of the model. The dashed lines of Figure 3 report the 90 percent posterior correlogram bands for the baseline model when employment data and the hours supply shock are incorporated in the estimation. The correlation of hours per worker and consumption growth is still too low relative to the data, while the correlation between employment and output growth is instead too high. Despite providing more information about labor market dynamics, the model still fails to ensure the positive correlation between hours and employment, and the $\beta_{\text{cov}}$ ranges from $-0.43$ to $0.37$ (see Table 2). In addition, this version of the model tends to overstate the importance of hours per worker relative to the data, as the posterior for $\beta_h$ ranges from $0.16$ to $0.76$, whereas its value is $0.18$ in the data. All in all, the baseline model—independently of the shocks considered or the observables included in the estimation—is unable to replicate satisfactorily the correlation structure between the aggregate macroeconomic series and the labor market variables.

The main issue is that hours per worker tends to be too countercyclical and too volatile in the model. Such counterfactual correlations reflect the negative comovement between the labor margins following standard supply and demand shocks, as well as in response to labor market shocks (either wage-bargaining shocks or hours-supply shocks). In particular, hours per worker fall in response to both positive technology and demand shocks, since the positive wealth effect reduces labor supply. By contrast, employment rises, since the surplus of hiring a worker increases in the presence of a nominal wage rigidity. Moreover, the response of hours per worker is as large as employment for several of the shocks we consider, reflecting the costless adjustment along the intensive margin.

To see this, Figure 4 reports the 90 percent posterior intervals for the impulse responses of output growth, employment, and hours per worker (solid lines denote the responses of the baseline model with eight shocks). We focus on the dynamics following innovations to aggregate TFP, investment-specific productivity, preferences, worker’s bargaining power and to the nominal interest rate, as these are the largest contributors to the variance of the growth rate of output, consumption, and investment. In all cases, responses are computed following a one standard deviation shock.

Four of the five shocks in Figure 4 depict negative comovement between the labor margins. The first column displays the responses following a positive shock to the growth rate of aggregate productivity. Total hours worked fall and the brunt of the impact adjustment is on the intensive
margin, as higher productivity induces a positive wealth effect that reduces labor supply. By contrast, employment is virtually unaffected on impact. The decline in hours per worker reduces the flow value of unemployment, leading to wage moderation. As a consequence, the surplus of hiring a worker increases, leading to higher employment after the first period. A similar mechanism is at work following an increase in the degree of impatience of households (column two). In this case, households substitute from investment to consumption. In turn, higher aggregate demand boosts employment, while the positive wealth effect crowds out hours per worker. The same logic applies to the monetary shock (column five), with the exception that the increase in the policy rate reduces aggregate demand, as the real interest rate increases. Finally, an exogenous increase in the workers' bargaining power (column four) directly reduces employment, since workers appropriate a larger share of the surplus through higher wages. Higher unemployment translates to lower aggregate demand and income, which once again increases hours supply. In addition, the decline in labor increases the marginal product of hours, other things equal, which also contributes to the positive response of the intensive margin.

In contrast to the previous disturbances, an increase in productivity specific to the production of the investment good displays positive comovement between the labor margins (column three). In this case, the wealth effect is negative, since households reduce consumption to finance higher investment. As a result, hours per worker increase, together with employment. A similar reasoning explains the positive comovement between the labor margins following an increase in government spending (not depicted), reflecting the crowding out of private consumption. Notice that the transmission of these two shocks rationalizes the ex ante positive comovement admitted in the prior predictive analysis discussed above, as for these shocks the strength and the direction of the wealth effect unambiguously favors an increase (reduction) in hours per worker at times of high (low) employment.

**Sensitivity Analysis** In Appendix II, we show that the specific value of the Frisch elasticity of labor supply is not central for the inability of the baseline model to account for the data. To illustrate this point, we estimated two alternative versions of the model with seven shocks, calibrating the Frisch elasticity either to values close to zero or to values used in the macroeconomic

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20The responses of macro aggregates and total hours following an increase in the disutility of hours worked $h_t$ (not depicted) are comparable to those following the bargaining power shock. In this case, the adjustment of the labor market margins are reversed, with hours per worker declining and employment rising.
Figure 4. Impulse response following a standard deviation innovation. Bands represent 90 percent confidence intervals. Solid lines denote the responses of the baseline model estimated with eight observables, while dashed lines correspond to the preferred framework.
literature. The labor margins still feature counterfactual dynamics in both cases.\footnote{When the Frisch elasticity is close to zero, hours volatility becomes counterfactually low. Alternatively, when the elasticity is consistent with a macro calibration (i.e. $\omega = 1$), the opposite holds. In both cases, the posterior estimates for the comovement between hours per worker and employment remain counterfactually low.} We also show the baseline model with seven shocks has difficulty in reproducing the data moments independent of which labor-market shock is included.

In addition, in Appendix H, we show that the inability of the baseline model with eight shocks to account for the data holds regardless of the data-measure considered for labor-market series and wages. We also consider an alternative determination of hours worked. We assume that firms have the right to manage hours. Under this alternative framework, firms choose $h_t$ to equate the marginal product of an hour worked to $w_t$. Our estimates show that this framework has a worse fit relative to the baseline model, implying more counterfactual labor-market dynamics. Finally, we estimated a real version of the baseline model, removing inflation and the nominal interest rate from the set of observables. We find the same counterfactual comovements in the labor margins even in this version of the baseline model. Moreover, not surprisingly, labor-market shocks become substantially more prominent in driving fluctuations in the margins of labor. We view these results as indirectly confirming that our results do not hinge on the specific source of employment volatility—i.e., whether or not the wage rigidity is the key driver of extensive-margin dynamics.

5 Alternative Model: Parametrized Wealth Effects and Costly Hours Adjustment

To address the shortcomings of the baseline model, we propose two modifications that reconcile the model with the data. First, we introduce preferences with a flexible parametrization of the strength of the short-run wealth effect on hours supply. In addition, to discipline the movement in hours worked, we allow for non-zero adjustment costs to the intensive margin, $\phi_h$. These two ingredients provide a parsimonious strategy to reproduce the correlations of labor market variables and macroeconomic series. We reference to this model as our preferred model.

Parametrized Wealth Effects on Labor Supply

We modify the period utility function to encompass a flexible parametrization of the strength of the short-run wealth effect on the labor supply. We consider the class of preferences first introduced
by Jaimovich and Rebelo (2009) (JR henceforth). Following Schmitt-Grohe and Uribe (2007), we modify the original JR specification to allow for internal consumption habit formation. The period utility function of the representative household now is given by:

\[
W_t \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} \beta_s \left[ \log \left( C_t - h_{t}\bar{C}_t - \bar{h}_t X_t \int_0^{L_t} \frac{h_{jt}^{1+\omega}}{1+\omega} dj \right) \right],
\]

(13)

where \( \gamma \in (0, 1] \) and \( X_t = (C_t - h_{C_t-1})^\gamma X_{t-1}^{1-\gamma} \). The parameter \( \gamma \) governs the magnitude of the short-run wealth effect on the labor supply. Abstracting from habit formation and time variation in the number of employed family members, when \( \gamma \to 0 \) this preference specification becomes identical to that of Greenwood et al. (1988). In this limiting case, the supply of labor is independent of the marginal utility of consumption. As a result, when \( \gamma \) is small, anticipated changes in income do not affect the current labor supply. As \( \gamma \) increases, the wealth elasticity of labor supply rises. In the polar case in which \( \gamma \) is unity, per-period utility becomes a product of habit-adjusted consumption and a function of hours worked.\(^{22}\)

The marginal disutility of an additional worker is now \( u_{h_{jt}} = \Psi_t^{-1} \bar{h}_t X_{jt}^{1+\omega} X_t / (1 + \omega) \), while the marginal disutility from labor supply is defined by:

\[
\frac{\partial u_{h_{jt}}}{\partial h_{jt}} \equiv \Psi_t^{-1} \bar{h}_t h_{jt}^{1+\omega} X_t,
\]

where \( \Psi_t \equiv C_t - h_{C_t-1} - \bar{h}_t X_t \int_0^{L_t} \left[ h_{jt}^{1+\omega} / (1 + \omega) \right] dj \). Thus, the marginal rate of substitution between hours and consumption for worker \( j \) continues to depend on aggregate variables, with the exception of hours worked, \( h_{jt} \). As a result, equation (7) implies that \( h_{jt} \) continues to depend only on aggregate conditions, so that \( h_{jt} = \bar{h}_t \) (and thus \( \tilde{h}_{jt} = \tilde{h}_t \)). Moreover, relative to the baseline model, the equilibrium wage differs only because of the different definitions of the value of the marginal product of labor and the flow value of unemployment implied by the parametrized wealth effect on the labor supply.

Overall, our modifications affect three equilibrium conditions defined in Table A.5 in the Appendix—equations (2), (14), and (15)—and three definitions in the same table—equations D.4-D.6.

\(^{22}\)Even with full risk-sharing within the household, the specification in equation (13) cannot be obtained by aggregating primitive utility functions for employed and unemployed workers. We have considered an alternative version of the model that features JR preferences for employed workers and a distinct utility function for unemployed family members. We then aggregate across agents, maintaining the assumption of full risk sharing within the household. Details are available upon request.
Estimation and Model Performance

We estimate the model with the same eight observables discussed above. For symmetry, we employ the same prior for hours adjustment costs as for investment adjustment costs, a normal distribution centered at 4 with a standard deviation of 1.5. This prior is diffuse enough to allow positive mass over a wide range of low and high adjustment cost values. We use a dogmatic prior for the parameter governing the strength of the wealth effect on labor supply, setting $\gamma = 0.01$. This value is sufficiently small to approach the limiting case of no wealth effects, consistent with the calibration of Jaimovich and Rebelo (2009). The priors for the remaining parameters are the same as those discussed in Section 4.

<table>
<thead>
<tr>
<th>Table 3: Model Fit Comparisons.</th>
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</thead>
<tbody>
<tr>
<td>Preferred Model</td>
</tr>
<tr>
<td>Baseline Model</td>
</tr>
<tr>
<td>Model w/o JR preferences</td>
</tr>
<tr>
<td>Model w/o costly hours adj.</td>
</tr>
</tbody>
</table>

Note: Log marginal data densities calculated using Geweke’s modified harmonic mean estimator; values are comparable conditional on observables, with different sets denoted by horizontal lines.

Table 3 reports the log marginal data densities and Bayes factors for the baseline and preferred models. Bayes factors quantify the relative support of two competing specifications given the observed data and are calculated from marginal data densities. Log marginal data densities are computed using Geweke’s (1999) modified harmonic mean estimator with a truncation parameter of 0.5. Kass and Raftery (1995) suggest that if twice the natural logarithm of the Bayes factor is greater than 2, then there is positive evidence in favor of the first model. Values greater than 10 suggest very strong evidence. The baseline model has a value substantially larger than 10, suggesting the data have strong preference for the model with JR preferences and hours adjustment.

We also have estimated a version of the model with a Beta prior for $\gamma$ centered at 0.5 with a standard deviation of 0.1. The posterior mean for $\gamma$ in this case is 0.16, outside the 90 percent prior bands. We have found that centering the prior of $\gamma$ at lower values results in lower posterior estimates and similar transmission mechanisms as our calibrated version.

Bayes factors convey the same kind of information as the Bayesian Information Criterion, see Kass and Raftery (1995). Model rankings are invariant to alternative truncation parameter choices. We restrict analysis to the parameter subspace that delivers a unique rational expectations equilibrium and denote this subspace as $\Theta_D$. In addition, we restrict parameters to ensure the steady-state wage lies within the feasible bargaining set. Let $I\{\theta \in \Theta_D\}$ be an indicator function that is one if the parameter vector $\theta$ is in the determinacy region and zero otherwise. Then, the joint prior distribution is defined as $p(\theta) = (1/c) \tilde{p}(\theta)I\{\theta \in \Theta_D\}$, where $c = \int_{\theta \in \Theta_D} \tilde{p}(\theta) d\theta$ and $\tilde{p}(\theta)$ denotes the joint prior density. Higher log marginal data density values imply greater fit.
costs. Furthermore, as shown in Table 3, including only JR preferences or hours adjustment costs improves the model fit relative to the baseline, although the inclusion of both is strongly preferred by the data over the individual alternatives.

Figure 5 plots the correlogram for several aggregate macroeconomic and labor market variables in the data (solid lines), as well as the 90 percent posterior intervals implied by both parameter and small sample uncertainty from this preferred model (dotted-dashed lines) and the baseline model with eight shocks (dashed lines). In almost all cases, the correlogram bands for the preferred model encapsulate the data counterparts, whereas the baseline model often fails to account for the cross-correlation structure of labor variables and macro aggregates. The preferred specification also implies variance decompositions of total hours more united with the data counterparts (see Table 2): $\beta_h$ ranges from 0.13 to 0.56, $\beta_L$ from 0.20 to 0.71, and $\beta_{cov}$ from $-0.05$ to 0.44. Finally, the last row of Figure 5 provides external model validation by displaying the correlograms for unemployment, a series not included among the estimation observables. Overall, the model accounts for unemployment dynamics, including comovements with macro data. In addition, relative to the baseline model, the preferred model accounts better for the comovement of unemployment with macro series. For instance, while the the bands for the correlation between unemployment and the intensive margin in the baseline model do not encompass the data at any horizon, the preferred model’s bands encompass the data for most horizons. In addition, the preferred model fits the correlation between unemployment and total hours and unemployment with itself better than the baseline model.25

The inclusion of JR preferences significantly improves the performance of the model. As described in the previous section, JR preferences reduce the strength of the short-run wealth effect on the labor supply. This mitigates the effect of variations in consumption on the marginal rate of substitution and makes hours per worker more responsive to changes in the value of the marginal product of hours. In turn, the increased comovement between hours per worker and employment strengthens the comovement of hours per worker with both output and investment. This also explains the data’s preference for positive hours’ adjustment costs, as they readjust the variability of the intensive margin relative to the data. By contrast, without hours’ adjustment costs, hours

25 The estimated preferred model is not only consistent with the labor margins emphasized in this paper, but also with the properties of the opportunity cost of employment emphasized in Chodorow-Reich and Karabarbounis (2016). To demonstrate this, we use the observables and the two-sided Kalman filter at the posterior mean estimates to generate smoothed, historical estimates of model variables, which by construction perfectly match employment and the other macro observables to the data. The model-implied series generate 1) the opportunity cost of employment to be more volatile than output and 2) procyclicality to the opportunity cost of employment, consistent with Chodorow-Reich and Karabarbounis (2016).
Figure 5. Correlograms from the data (blue solid lines) and 90 percent posterior intervals from 1) the preferred model (red dotted lines) and 2) the baseline model (black dashed lines).
per worker tend to be too volatile, implying that hours per worker and employment contribute roughly equally to total hours variation (see Table A.4 in the Appendix). Note that as reported in the Appendix Table A.2, the estimated persistence and standard deviations of innovations are similar across the baseline and preferred specifications, suggesting that the improved fit can be traced to an improvement in the propagation mechanism rather than to different estimates of the shock processes.

To substantiate this intuition, we study the propagation mechanisms of the same shocks previously discussed for the baseline model. The dashed lines of Figure 4 report the 90 percent confidence intervals for the impulse responses of output growth, employment, and hours per worker in the preferred model. In this case, all shocks depicted, except the bargaining power shock, induce positive comovement between the labor margins. For instance, following a shock to technology, JR preferences reduce the wealth effect on the labor supply relative to the baseline preferences, causing hours per worker to drop less on impact. This, in turn, reduces its effect on the firm’s surplus, leading employment to decline on impact as well. A similar mechanism explains the positive comovement following a shock to preferences or monetary policy. In summary, standard supply and demand shocks—technology, investment-specific, preference, and monetary policy shocks, as well as price markup and government spending shocks (not depicted)—now induce positive comovement between the labor margins in the preferred model. By contrast, labor-market shocks—shocks to workers’ bargaining power and hours supply shocks (not depicted)—continue to induce negative comovement (as in the baseline model). Although the weaker wealth effect dampens the increase in hours per worker, the rise in the value of the marginal product of hours brought about by the reduction in employment still results in a positive response of the intensive margin.

*The Role of Hours Supply Shocks*

While parametrized wealth effects and hours adjustment costs are key ingredients for the model to reproduce the empirical covariances of labor market variables, hours supply shocks remain an important contributor for the variance of hours per worker—\( \tilde{h}_t \) accounts for about 60 percent of the volatility of \( h_t \) (see Appendix F). A natural question concerns the interpretation of hours supply shocks in the context of the model. One possibility is that \( \tilde{h}_t \) simply reflects measurement error. To rule out this hypothesis, we estimate an alternative version of the model that additionally allows for measurement error in each observable. Even in this case, hours supply shocks still remain an important contributor to fluctuations in hours per worker (results available upon request).
A closer look at the model’s equilibrium conditions presents a simple structural interpretation for $\bar{h}_t$. Consider the log-linear approximation of the intratemporal condition for optimality in hours (normalizing to one steady-state hours per worker):

$$\hat{\beta}_t - \frac{1}{\Psi} \left[ \dot{C}_t - h_t C_{t-1} \frac{1}{g_A} \left( \hat{C}_{t-1} - \hat{g}_{At} \right) - \frac{LX}{1 + \omega} \left( \hat{L}_t + (1 + \omega) \hat{h}_t + \hat{X}_t \right) + \hat{h}_t + \hat{X}_t - \hat{u}_C t \right]$$

$$= \hat{\phi}_t + \alpha \left( \hat{u}_K t + \hat{K}_t - \hat{g}_{At} - \hat{L}_t - \hat{h}_t \right) - \phi_{h} \hat{h}_t - \left( \omega + \frac{XL}{(1 + \omega)\Psi} \right) \hat{h}_t,$$

where hats denote log-deviations. The right-hand side of this equation shows that $\bar{h}_t$ acts as a time-varying shifter of the marginal product of one hour worked, consistent with the empirical observation that changes or differences in working hours do not entail the same changes or differences in effective labor input (Pencavel, 2015). Thus, $\bar{h}_t$ captures cyclical fluctuations in unobservable utilization of hours per worker, reflecting variations in unobserved worker effort (see, for instance, Kimball et al., 2006). In addition, $\bar{h}_t$ captures exogenous fluctuations in the value of nonworking time, i.e., the value of the opportunity cost of employment.

6 Hours and Employment in Post-War U.S. Business Cycles

We now use the preferred model to empirically study the cyclical behavior of hours and employment in U.S. data. Consistent with the empirical evidence presented in Section 2, we focus on U.S. business cycle recessions and recoveries—i.e., the progression of the economy after having hit the trough of a recession.

The impulse-response analysis discussed in the previous section rationalizes the observed time-varying comovement between the margins of labor adjustment. When recoveries feature a prominent role for standard supply and demand shocks, hours per worker and employment comove positively. By contrast, when labor market shocks have a larger role, the adjustment along the two margins displays a much lower correlation. The historical decompositions of employment and hours per worker from the preferred model (presented in Appendix G) show that structural innovations responsible for employment and hours per worker fluctuations in recoveries are consistent with these comovements. For example, recessions and recoveries of 1970, 1975, and 1982 are predominantly driven by investment-specific shocks, while those in 1991 and 2001 feature a more significant role

\[26\] Marchetti and Nucci (2014) document a hump-shaped profile of labor effort at business cycle frequencies. Notice that $\bar{h}_t$ may also capture in reduced-form other unmodeled features of hours adjustment such as overtime hours. A formal assessment of the quantitative importance of this alternative interpretation is precluded by the absence of economy-wide data for overtime hours in the U.S. economy.
for labor-market shocks, a finding consistent with Aaronson et al. (2004).

Our structural model also provides an ideal laboratory to quantify the contribution of the intensive margin to total hours. Intuitively, the overall effect on total hours comes from both a direct and an indirect effect. The direct effect accounts for the variation in total hours stemming from adjustments in hours per worker for a given level of employment. The indirect effect accounts for the impact that fluctuations in the intensive margin have on new matches (and thus employment) in response to aggregate shocks. Thus, to evaluate the contribution of direct and indirect effects, we conduct the following Bayesian counterfactual experiment. First, we use the posterior mean estimates of the preferred model and the two-sided Kalman filter to construct smoothed estimates of the structural shocks and model variables. We then construct a counterfactual time series where hours are held constant at their steady-state value and compare the actual path to the hypothetical one where hours per worker are constant.27

Figure 6 contrasts the actual values of total hours, hours per worker, and employment (solid lines) with the model counterfactual values (dashed lines). Since total hours and employment are observables and there is no measurement error, the smoothed estimates of these variables from the two-sided Kalman filter, as well as hours per worker, perfectly match the data by construction. The figure shows that adjustment in hours per worker has sizable effects on the recovery of total hours (up to 3.5 percentage points in the recovery of 1982, where the decline in total hours is halved with fixed hours). In particular, lack of adjustment along the intensive margin would have mitigated employment losses during recessions and resulted in quicker employment recoveries, i.e., hours adjustment contributes positively to employment volatility during post-war U.S. recessions and recoveries. This result is confirmed when considering unconditional volatilities. In the counterfactual economy, the standard deviations of employment and total hours respectively drop by 13 and 40 percent over the sample. Two-thirds of the lower volatility in total hours is accounted for by the lack of adjustment in hours-per-worker while the remaining part depends on the indirect effect of the intensive margin on employment.

What accounts for the importance of the indirect effect of hours per worker on employment? Figure 7 plots the differences in the historical decompositions of employment from the preferred and counterfactual models. As seen in the figure, investment specific shocks account for large differences in the dynamics of the two economies in the recoveries of 1970, 1975 and 2001. Shocks to workers’

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27 Results are unaffected if hours per worker are held constant at the beginning of the trough of each recession (rather than at the beginning of the sample).
bargaining power are more important in the recovery of 1991, while the lack of hours supply shocks dominates in 1982.

To understand the different transmission of these shocks with and without the intensive-margin adjustment, Figure 8 plots the impulse responses in the counterfactual economy with constant hours (dashed lines) relative to the preferred model responses (solid lines). An increase in investment-specific productivity remains expansionary in the counterfactual economy. However, the absence of hours adjustment implies a larger response of effective capital and a much lower response of total hours. Since this shock induces positive comovement between the labor margins, the expected present discounted value of new matches rises less in response to positive shocks when hours are constrained and do not increase. Moreover, achieving the same adjustment of total hours would require additional vacancy posting in the counterfactual economy, which is costly due to congestion externalities. All else equal, this decreases firms' incentives to adjust employment, mitigating employment's response. Accordingly, firms raise effective capital more to meet a given demand.\footnote{Notice that, in general equilibrium, a second channel occurs through relative price adjustments. Higher demand for effective capital raises the rental rate of capital, giving an incentive to producers to substitute capital with labor (i.e., to hire more workers). This feedback effect is stronger for shocks that are more persistent, since the}
In the case of an exogenous increase in the workers’ bargaining power, employment falls less when hours per worker cannot increase, since firms can no longer substitute across the labor margins. Finally, with constant hours per worker, the transmission of the hours supply shock is directly eliminated, contributing to the dampened decline in total hours.

7 Conclusions

Using U.S. data, we estimate a quantitative search-and-matching model augmented with endogenous fluctuations in hours per worker. Likelihood maximization with both macroeconomic and labor data renders structural parameter estimates that cannot jointly reproduce the comovement of the labor margins with themselves and with macro data. Two proposed modifications reconcile the model with the data: adjustment costs to the intensive margin and a flexible parametrization of the strength of the short-run wealth effect on hours supply, as first introduced by Jaimovich and Rebelo (2009).

We use the modified model to structurally assess the contribution of the adjustment in hours present discounted value of job creation increases more in this case. In turn, the strength of the general equilibrium effect accounts for the responses to preference and markup shocks in Figure 8, as these shocks have high estimated persistence.
Figure 8. Impulse response following a one standard deviation innovation at the posterior mean. Solid lines denote the responses of the preferred model, while dashed lines correspond to the counterfactual economy without adjustment in hours per worker.
per worker to aggregate dynamics. The model offers a structural explanation for the observed
time-varying comovement between the labor margins (i.e., positive or negative) across post-war
U.S. recessions and recoveries. In these episodes, the contribution of hours per worker to total
hours is quantitatively significant, with a notable component stemming from its indirect effect on
employment’s adjustment.

While we estimate the model on U.S. data, the results of the paper are broader in scope, as
the inability of the baseline model to account for the margins of labor adjustment is not limited to
the U.S. economy. For instance, as documented by Ohanian and Raffo (2012), hours and employ-
ment positively comove in several economies (for instance, in the U.K. and Canada). In addition,
parametrized wealth effects and costly hours’ adjustment introduce enough flexibility to match the
broad array of empirical covariances of hours per worker and employment observed in the cross-
section of countries. Discerning the role of intensive-margin adjustment for other countries is an
important avenue for future research.

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