Online Appendix to “Hours and Employment Over the Business Cycle: A Structural Analysis”
Not For Publication

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Abstract
This Appendix gathers supplementary material to Cacciatore, Fiori, and Traum (2019).

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A Data Description

We consider three alternative sets of labor market variables: CES, CPS, and the hours series constructed by Smets and Wouters (2007). The estimation in the main text uses the first set, which the literature comparing employment measures in jobless recoveries suggests preference for (see Bachmann (2012) for a review of the literature). For robustness, we consider in this section also the two alternative data sets. We apply the following transformation to Total Hours and Employment: \( \ln \left( \frac{X}{\text{Pop}} \right) \times 100 \) where \( \text{Pop} \) is the civilian noninstitutional population (series LNU00000000 of the BLS). We apply the following transformation to hours per worker: \( \ln(h) \times 100 \).


Total Hours, \( TH \) is economy-wide total hours measure of the BLS, taken from www.bls.gov/lpc/special_requests/us_total_hrs_emp.xlsx.

Employment, \( L \) is the economy-wide employment series of the BLS (from same source as total hours).

Hours, \( h \) is average weekly hours calculated as \( (TH/L)/52 \).


Total Hours, \( TH \) is an economy-wide measure constructed as in Ramey (2012). The total hours series of Cociuba et al. (2012), which is constructed from the BLS’ CPS data for all industries, combined with total hours of the armed forces, taken from www.bls.gov/lpc/special_requests/us_total_hrs_emp.xlsx.

Employment, \( L \) is an economy-wide employment series constructed from combining CPS employment for all industries with armed forces employment (from same sources as total hours).

Hours, \( h \) is average weekly hours calculated as \( (TH/L)/52 \).

3. Smets-Wouters (SW) data.

Hours, \( h \) is defined as the index for nonfarm business, all persons, average weekly hours duration, 2009 = 100, seasonally adjusted (from the Major Sector Productivity and Cost series PRS85006023 of the BLS).
**Employment,** \(L\) is civilian employment for all industries for ages sixteen years and over, seasonally adjusted (from the CPS series LNS12000000Q of the BLS).

**Total Hours,** \(TH\) is calculated as \(h \times L\).

In addition to the labor market variables, the following data are used for estimation. Unless otherwise noted, data are from the National Income and Product Accounts tables of the Bureau of Economic Analysis.

**GDP.** Gross domestic product (Table 1.1.5 line 1). Output \((y)\) growth is

\[
100 \left[ \ln \left( \frac{y_t}{gdpp_t pop_t} \right) - \ln \left( \frac{y_t-1}{gdpp_{t-1} pop_{t-1}} \right) \right]
\]

where \(gdpp\) is the GDP deflator (Table 1.1.4, line 1) and \(Pop\) is the civilian noninstitutional population (series LNU00000000 of the BLS).

**Consumption.** Total personal consumption expenditures on nondurables and services (Table 1.1.5, lines 5 and 6). Consumption \((c)\) growth is

\[
100 \left[ \ln \left( \frac{c_t}{gdpp_t pop_t} \right) - \ln \left( \frac{c_{t-1}}{gdpp_{t-1} pop_{t-1}} \right) \right]
\]

where \(gdpp\) is the GDP deflator (Table 1.1.4, line 1) and \(Pop\) is the civilian noninstitutional population (series LNU00000000 of the BLS).

**Investment.** Gross private domestic investment (Table 1.1.5, line 7) and personal consumption expenditures on durables (Table 1.1.5, line 4). Investment \((i)\) growth is

\[
100 \left[ \ln \left( \frac{i_t}{gdpp_t pop_t} \right) - \ln \left( \frac{i_{t-1}}{gdpp_{t-1} pop_{t-1}} \right) \right]
\]

where \(gdpp\) is the GDP deflator (Table 1.1.4, line 1) and \(Pop\) is the civilian noninstitutional population (series LNU00000000 of the BLS).

**Wage Rate.** The wage rate \(w\) is the index for hourly compensation for nonfarm business, all persons, 2009 = 100 (from the Major Sector Productivity and Cost series PRS85006103 of the BLS). Wage growth is

\[
100 \left[ \ln \left( \frac{w_t}{gdpp_t} \right) - \ln \left( \frac{w_{t-1}}{gdpp_{t-1}} \right) \right]
\]

where \(gdpp\) is the GDP deflator (Table 1.1.4, line 1).

**Inflation.** The gross inflation rate is the log first difference of the GDP deflator (Table 1.1.4, line 1).
Interest Rate. The nominal interest rate is the average of daily figures of the Federal Funds Rate (from the Board of Governors of the Federal Reserve System) divided by 4.

Inflation and the interest rate are demeaned, while total hours and employment are linearly detrended. Figure A.1 illustrates our preference for linearly detrended data. Over the sample period, hours per worker exhibits a downward trend while employment exhibits an upward trend. When these (logged) variables are linearly detrended, their sum almost perfectly matches the original, demeaned total hours series (their correlation is 0.9999). Thus, the linear filtering appears to account for the low-frequency structural features of employment and hours per worker while preserving the original properties of the total hours series. In contrast, HP filtered hours per worker and employment change the properties of a total hours measure. GDP, consumption, investment, and wages are neither demeaned nor detrended.
Observables are linked to model variables in the following manner:

$$
\begin{bmatrix}
\text{GDP}_t \\
\text{Cons}_t \\
\text{Inv}_t \\
\text{Wage}_t \\
\text{TotalHour}_t \\
\text{Emp}_t \\
\text{Infl}_t \\
\text{FedFunds}_t
\end{bmatrix}
= 
\begin{bmatrix}
100 \log \bar{g}_A \\
100 \log \bar{g}_A \\
100 \log \bar{g}_A \\
100 \log \bar{g}_A \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\dot{y}_t - \hat{y}_{t-1} + \hat{g}_A \\
\dot{c}_t - \hat{c}_{t-1} + \hat{g}_A \\
\dot{i}_t - \hat{i}_{t-1} + \hat{g}_A \\
\dot{w}_t - \hat{w}_{t-1} + \hat{g}_A
\end{bmatrix}
+ 
\begin{bmatrix}
\bar{T}H_t \\
\hat{L}_t \\
\hat{\pi}_t \\
\hat{R}_t
\end{bmatrix}
$$

**Hours and Employment with Alternative Data**

We document the robustness of the results of Section 2 of the main paper to alternative measures of the labor market variables. Table A.1 displays the shares of hours per worker and employment for the variance of total hours for the two alternative labor market data sets described above, based on CPS data and Smets and Wouters (2007) observables. The shares are calculated after applying various transformations on the data and for two alternative sample periods. In all but one case, the covariance of hours per worker and employment is positive. Hours per worker accounts for 15-48% of the variance of total hours.

Figure A.2 reports the time-varying comovement in recession-recovery episodes using alternative detrending methods using the CES dataset. In addition to the linear detrending procedure, we apply a HP filter with smoothing parameters of $10^5$ and a band pass filter as in Christiano and Fitzgerald (2003). Results also hold for the alternative measures of labor-market variables (not pictured).

**B Privately Efficient Hours**

Here we show that the optimality condition in hours worked presented in the main text is implied by joint-surplus maximization by the firm and the worker. Assume that the firm and the worker choose hours per worker to maximize the total surplus of the match:

$$
h_{jt} = \arg \max \left\{ S^f_{jt} + S^w_{jt} \right\},
$$
**TABLE A.1: Components of the Variance of Total Hours for Alternative Data Sources**

<table>
<thead>
<tr>
<th>Filtering</th>
<th>( \beta_{\text{cov}, h} )</th>
<th>( \beta_{\text{cov}, L} )</th>
<th>( \beta_h )</th>
<th>( \beta_L )</th>
<th>( \beta_{\text{cov}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{cov}(T H_t, h_t) ) ( \text{var}(T H_t) ) ( \text{var}(h_t) ) ( \text{var}(L_t) ) ( 2\text{cov}(h_t, L_t) ) ( \text{var}(T H_t) ) ( \text{var}(T H_t) ) | 1965:Q1-2007:Q4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPS data</td>
<td>Demeaned</td>
<td>0.15</td>
<td>0.85</td>
<td>0.07</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>Linear</td>
<td>0.34</td>
<td>0.66</td>
<td>0.15</td>
<td>0.46</td>
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<tr>
<td></td>
<td>HP</td>
<td>0.28</td>
<td>0.72</td>
<td>0.12</td>
<td>0.56</td>
</tr>
<tr>
<td>SW data</td>
<td>Linear</td>
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<td>0.52</td>
<td>0.39</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>HP</td>
<td>0.28</td>
<td>0.72</td>
<td>0.14</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>1965:Q1-2014:Q4</td>
<td>0.17</td>
<td>0.83</td>
<td>0.09</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>Linear</td>
<td>0.29</td>
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<td>HP</td>
<td>0.31</td>
<td>0.69</td>
<td>0.14</td>
<td>0.51</td>
</tr>
<tr>
<td>SW data</td>
<td>Linear</td>
<td>0.16</td>
<td>0.84</td>
<td>0.24</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>HP</td>
<td>0.27</td>
<td>0.73</td>
<td>0.13</td>
<td>0.58</td>
</tr>
</tbody>
</table>

*Data from 1965:1-2011:IV.

Decomposition of Total Hours in U.S. Recoveries

![Graph showing GDP Growth and decomposition of total hours](image)

Figure A.2. CES labor market variables. Linear denotes linearly detrended data; HP denotes Hodrick-Prescott filtered series with smoothing parameter of 10^5; BP denotes series filtered with the Christiano and Fitzgerald (2003) procedure with frequency 2.32.
where $S^f_{jt}$ denotes the firm surplus:

$$S^f_{jt} = (1 - \alpha) \varphi_t \left( \frac{K_{jt}}{A_t \tilde{h}_{jt} L_{jt}} \right)^{\alpha} \tilde{A}_t \tilde{h}_{jt} - \frac{w^n_{jt} \tilde{h}_{jt}}{P_t} - \Gamma_{w,j,t} + E_t \beta_{t,t+1} (1 - \lambda) S^f_{jt+1},$$

and $S^w_{jt}$ denotes the worker surplus:

$$S^w_{jt} = \frac{w^n_{jt}}{P_t} \tilde{h}_{jt} - \frac{u_{h_{jt}}}{u_{C_t}} + E_t \left[ \beta_{t,t+1} (1 - \lambda) S^w_{jt+1} \left( 1 - \frac{M_{t+1}}{U_{t+1}} \right) \right]. \quad (A-1)$$

The variables appearing above are defined as in the main text—in particular, $u_{h_{jt}}$ denotes the marginal disutility of an additional worker, and $u_{C_t}$ is the marginal utility of consumption. Using the first-order condition for capital:

$$\varphi_t \alpha \left( \frac{K_{jt}}{A_t L_{jt} \tilde{h}_{jt}} \right)^{\alpha - 1} = r_{K_t},$$

the firm surplus can be written as

$$S^f_{jt} = (1 - \alpha) \varphi_t \left( \frac{r_{K_t}}{\varphi_t^{\alpha}} \right)^{\frac{\alpha}{\alpha - 1}} \tilde{A}_t \tilde{h}_{jt} - \frac{w^n_{jt}}{P_t} \tilde{h}_{jt} - \Gamma_{w,j,t} + E_t \beta_{t,t+1} (1 - \lambda) S^f_{jt+1}. \quad (A-2)$$

The joint surplus is then given by

$$S^f_{jt} + S^w_{jt} = (1 - \alpha) \varphi_t \left( \frac{r_{K_t}}{\varphi_t^{\alpha}} \right)^{\frac{\alpha}{\alpha - 1}} \tilde{A}_t \tilde{h}_{jt} - \frac{w^n_{jt}}{P_t} \tilde{h}_{jt} - \Gamma_{w,j,t} + E_t \beta_{t,t+1} (1 - \lambda) S^f_{jt+1}$$

$$- \frac{u_{h_{jt}}}{u_{C_t}} + E_t \left[ \beta_{t,t+1} (1 - \lambda) S^w_{jt+1} \left( 1 - \frac{M_{t+1}}{U_{t+1}} \right) \right].$$

The first-order condition with respect to $h_{jt}$ implies

$$(1 - \alpha) \varphi_t \left( \frac{r_{K_t}}{\varphi_t^{\alpha}} \right)^{\frac{\alpha}{\alpha - 1}} \tilde{A}_t \Delta_{h_{jt}} = \frac{\partial u_{h_{jt}} / \partial h_{jt}}{u_{C_t}}.$$
The optimal choice of hours per worker implies:

\[ \frac{\partial u_{h_jt}/\partial h_{jt}}{u_{Ct}} = (1 - \alpha) \varphi_t \left( \frac{r_{Kt}}{\varphi_t} \right)^{\alpha-1} \tilde{A}_t \Delta_{h_{jt}}, \quad (A-3) \]

as in the main text. Notice that, up to a first-order approximation, \( \tilde{h}_{jt} = h_{jt} \). Moreover, equation A-3 shows that \( h_{jt} \) only depends on aggregate conditions, i.e., \( h_{jt} = h_t \) is invariant to the scale of the firm. Finally, \( h_{jt} \) does not directly depend on the hourly wage \( w_{jt} \).

C Wage Bargaining

The firm and worker maximize the Nash product

\[ \left( S^f_t \right)^{1-h_t} \left( S^w_t \right)^{h_t}, \]

where, as detailed in the main text:

\[ S^f_t = (1 - \alpha) \varphi_t \left( \frac{r_{Kt}}{\varphi_t} \right)^{\alpha-1} \tilde{A}_t h_t - \frac{w^n_t h_t}{P_t} - \phi^w \tilde{A}_t \left( \frac{w^n_t \pi^w_n - \pi^w_i - \bar{g} A}{w^n_{t-1} \pi^w_i \pi^w - \bar{g} A} \right)^2 + E_t \beta_{t,t+1} (1 - \lambda) S^f_{t+1} \]

and

\[ S^w_t = \frac{w^n_t h_t - b \tilde{A}_t - \frac{u_{ht}}{u_{Ct}} + (1 - \lambda) E_t \left( 1 - \frac{M_{t+1}}{U_{t+1}} \right)}{P_t} \]

The first-order condition with respect to \( w^n_t \) implies

\[ (1 - \bar{h}_t) S^w_t \frac{\partial S^f_t}{\partial w^n_t} + \bar{h}_t S^f_t \frac{\partial S^w_t}{\partial w^n_t} = 0, \quad (A-4) \]

where

\[ \frac{\partial S^f_t}{\partial w^n_t} = -\frac{h_t}{P_t} - \phi^w \tilde{A}_t \left( \frac{w^n_t \pi^w_n - \pi^w_i - \bar{g} A}{w^n_{t-1} \pi^w_i \pi^w - \bar{g} A} \right) + (1 - \lambda) E_t \left( \beta_{t,t+1} \frac{\partial S^f_{t+1}}{\partial w^n_t} \right) \quad (A-5) \]

and

\[ \frac{\partial S^w_t}{\partial w^n_t} P_t = h_t. \]

(Notice that we have used the fact that \( \partial w^n_t / \partial h_t = 0 \), which stems from equation (A-3).) Moreover, notice that

\[ \frac{\partial S^f_{t+1}}{\partial w^n_t} = \phi^w \tilde{A}_{t+1} \left( \frac{w^n_{t+1} \pi^w_n - \pi^w_i - \bar{g} A}{w^n_t \pi^w_i \pi^w - \bar{g} A} \right) \left( \frac{w^n_t \pi^w_n - \pi^w_i - \bar{g} A}{w^n_t \pi^w_i \pi^w - \bar{g} A} \right)^2. \quad (A-6) \]
By inserting (A-6) into (A-5), we finally obtain:

$$\frac{\partial S^f_t}{\partial w_t} P_t = -h_t - \phi^w \tilde{A}_t \left( \pi_{wt} \pi^{-1} - \bar{\pi}_A \right) \frac{\pi^{-1} - \bar{\pi}}{w_{t-1}}$$

$$+ \phi^w (1 - \lambda) E_t \left[ \beta_{t+1} \tilde{A}_{t+1} \left( \pi_{wt+1} \pi^{-1} - \bar{\pi}_A \right) \frac{\pi^{-1} - \bar{\pi}}{w_t} \right],$$

(A-7)

where $w_t \equiv w_t^v/P_t$.

Finally, let

$$\eta_{wt} = \frac{\hat{\eta}_t \hat{S}^w_t}{\hat{\eta}_t \hat{S}^w_{wt} - (1 - \hat{\eta}_t) \hat{S}^f_{wt}}.$$ 

The latter means that

$$1 - \eta_{wt} = \frac{-(1 - \hat{\eta}_t) \hat{S}^f_{wt}}{\hat{\eta}_t \hat{S}^w_{wt} - (1 - \hat{\eta}_t) \hat{S}^f_{wt}}.$$ 

Using the above expression, the sharing rule in (A-4) can be written more compactly as

$$(1 - \eta_{wt}) S^w_t = \eta_{wt} S^f_t,$$

where $\eta_{wt}$ measures the effective bargaining power of the worker and $1 - \eta_{wt}$ is the effective bargaining power of the firm. Notice that, using equation (A-7), the effective bargaining power of the worker can be written as

$$\eta_{wt} = \frac{\tilde{\eta}_t h_t}{\tilde{\eta}_t h_t - (1 - \tilde{\eta}_t) \left[ -h_t - \phi^w \tilde{A}_t \left( \pi_{wt} \pi^{-1} - \bar{\pi}_A \right) \frac{\pi^{-1} - \bar{\pi}}{w_{t-1}} 
+ \phi^w E_t \beta_{t+1} (1 - \lambda) \tilde{A}_{t+1} \left( \pi_{wt+1} \pi^{-1} - \bar{\pi}_A \right) \frac{\pi^{-1} - \bar{\pi}}{w_t} \right]}.$$ 

When $\phi^w = 0$, the expression above simplifies to $\eta_{wt} = \tilde{\eta}_t$.

D Balanced Growth Path and Log-linearized Model

All the non-stationary variables are normalized by the level of labor productivity, i.e., $X_t/\tilde{A}_t$ (with the exception of the marginal utility of consumption, which is normalized by $u_C \tilde{A}_t$). In order to economize on notation, we do not change notation for those variables. Table A.5 below describes the stationary version of the baseline model, while Table A.6 presents the stationary version of the
alternative model’s equations.

We log-linearize the model around the deterministic balanced growth path. Below, endogenous variables that appear without a time subscript denote constant normalized variables. Notice that, in the deterministic steady state:

\[ \bar{Z} = \bar{\beta} = \bar{p}_K = \bar{\zeta}_K = \bar{\pi}_C = u_K = 1, \]

while \( \pi_w = g_A \). Finally, let \( \hat{x}_t \equiv \frac{dx_t}{x} \sim \log(x_t) - \log(x) \). Table A.7 presents the log-linearized equations. Finally, notice that, starting from the stationary log-linear system, we recover a given non-stationary variable \( x_t^L \) by constructing \( x_t^L = (e^{\hat{x}_t} + x) \bar{A}_t \). The growth rate of the non-stationary variable is then obtained as follows:

\[ \Delta x_t^L \equiv \log(x_t^L) - \log(x_{t-1}^L) = \hat{x}_t - \hat{x}_{t-1} + \hat{g}_A t + \log(g_A). \]

E Estimation Procedure and Posterior Estimates

To estimate the model, we construct the parameters’ posterior distribution using Bayesian inference methods. The posterior distribution is a combination of a prior density for the parameters and the likelihood function, evaluated using the Kalman filter. We take 1.5 million draws from the posterior distribution using the random walk Metropolis-Hastings algorithm. For inference, we discard the first 500,000 draws and keep one every 50 draws to remove some correlation of the draws.\(^1\)

Table A.2 reports the posterior estimates of the baseline model presented in Section 3 of the main paper. As discussed in the paper, we estimate two versions of this model. The first includes seven observables and seven shocks: TFP, investment, preference, government spending, interest rate, price markup, and bargaining shocks. Parameter estimates from this version are listed under the column “7 shocks.” The second version includes an additional observable, employment, and an additional labor market shock, the hours-supply shock \( \bar{h}_t \). Parameter estimates from this version are listed in the column “Baseline” under the headings “8 shocks” in Table A.2.

The columns under the heading “7 shocks” in Table 2 list the posterior mean and 90 percentile estimates of the baseline model estimated with seven shocks and observables, while the columns under the headings ‘8 shocks’ and ‘Baseline Model’ list the estimates with eight shocks and observ-

\(^1\)We set the step size to ensure the acceptance rate is in the range of 20 to 40 percent for all variations of the estimated model. Convergence diagnostics include cumulative sum of draws (cumsum) statistics and Geweke’s Separated Partial Means (GSPM) test. Results are available from the authors.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>7 shocks</th>
<th>Posterior</th>
<th>8 shocks</th>
<th>8 shocks</th>
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<td>Model</td>
<td>Mean</td>
<td>90% Int</td>
<td>Mean</td>
<td>90% Int</td>
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<tr>
<td>Preferences</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$h_C$, habit formation</td>
<td>B</td>
<td>0.5 0.1</td>
<td>0.79 [0.73, 0.83]</td>
<td>0.68 [0.63, 0.72]</td>
<td>0.79 [0.73, 0.84]</td>
</tr>
<tr>
<td>$w$, inverse Frisch</td>
<td>G</td>
<td>2 0.5</td>
<td>3.44 [2.49, 4.33]</td>
<td>6.98 [5.83, 8.24]</td>
<td>3.74 [1.94, 5.68]</td>
</tr>
<tr>
<td>$\zeta$, capital utilization</td>
<td>B</td>
<td>0.5 0.1</td>
<td>0.79 [0.73, 0.83]</td>
<td>0.68 [0.63, 0.72]</td>
<td>0.79 [0.73, 0.84]</td>
</tr>
<tr>
<td>$\theta$, workers bargaining power</td>
<td>B</td>
<td>0.5 0.1</td>
<td>0.79 [0.73, 0.83]</td>
<td>0.68 [0.63, 0.72]</td>
<td>0.79 [0.73, 0.84]</td>
</tr>
<tr>
<td>$\tau$, convexity vacancy cost</td>
<td>G</td>
<td>2 0.5</td>
<td>3.44 [2.49, 4.33]</td>
<td>6.98 [5.83, 8.24]</td>
<td>3.74 [1.94, 5.68]</td>
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<td>B</td>
<td>0.5 0.1</td>
<td>0.79 [0.73, 0.83]</td>
<td>0.68 [0.63, 0.72]</td>
<td>0.79 [0.73, 0.84]</td>
</tr>
<tr>
<td>$\xi^c$, price stickiness</td>
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<td>0.5 0.1</td>
<td>0.79 [0.73, 0.83]</td>
<td>0.68 [0.63, 0.72]</td>
<td>0.79 [0.73, 0.84]</td>
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<tr>
<td>$\phi_{\pi}$, interest resp. to inflation</td>
<td>G</td>
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<td>0.05 [0.02, 0.09]</td>
<td>0.10 [0.06, 0.14]</td>
<td>0.07 [0.03, 0.12]</td>
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<tr>
<td>$\phi_Y$, interest resp. to Y gap</td>
<td>N</td>
<td>0.13 0.05</td>
<td>0.34 [0.28, 0.40]</td>
<td>0.31 [0.25, 0.36]</td>
<td>0.28 [0.23, 0.34]</td>
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<tr>
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<td>0.73 [0.68, 0.77]</td>
<td>0.75 [0.70, 0.79]</td>
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<td>Shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{\Delta A}$, technology</td>
<td>B</td>
<td>0.5 0.2</td>
<td>0.14 [0.05, 0.24]</td>
<td>0.07 [0.02, 0.13]</td>
<td>0.10 [0.03, 0.19]</td>
</tr>
<tr>
<td>$\rho_{\beta_{\Delta}}$, preference</td>
<td>B</td>
<td>0.5 0.2</td>
<td>0.14 [0.05, 0.24]</td>
<td>0.07 [0.02, 0.13]</td>
<td>0.10 [0.03, 0.19]</td>
</tr>
<tr>
<td>$\rho_{PK}$, investment</td>
<td>B</td>
<td>0.5 0.2</td>
<td>0.84 [0.78, 0.9]</td>
<td>0.20 [0.10, 0.30]</td>
<td>0.20 [0.11, 0.30]</td>
</tr>
<tr>
<td>$\rho_{\theta_{\gamma}}$, price markup</td>
<td>B</td>
<td>0.5 0.2</td>
<td>0.84 [0.78, 0.9]</td>
<td>0.20 [0.10, 0.30]</td>
<td>0.20 [0.11, 0.30]</td>
</tr>
<tr>
<td>$\rho_{g}$, govt cons</td>
<td>B</td>
<td>0.5 0.2</td>
<td>0.84 [0.78, 0.9]</td>
<td>0.20 [0.10, 0.30]</td>
<td>0.20 [0.11, 0.30]</td>
</tr>
<tr>
<td>$\rho_{\eta_{\theta}}$, bargaining</td>
<td>B</td>
<td>0.5 0.2</td>
<td>0.84 [0.78, 0.9]</td>
<td>0.20 [0.10, 0.30]</td>
<td>0.20 [0.11, 0.30]</td>
</tr>
<tr>
<td>$\rho_{\tau_{\eta}}$, monetary shock</td>
<td>B</td>
<td>0.5 0.2</td>
<td>0.84 [0.78, 0.9]</td>
<td>0.20 [0.10, 0.30]</td>
<td>0.20 [0.11, 0.30]</td>
</tr>
<tr>
<td>$100\sigma_{\Delta A}$, technology</td>
<td>IG</td>
<td>0.5 1</td>
<td>0.83 [0.75, 0.92]</td>
<td>1.01 [0.92, 1.11]</td>
<td>1.07 [0.97, 1.19]</td>
</tr>
<tr>
<td>$100\sigma_{\beta_{\Delta}}$, preference</td>
<td>IG</td>
<td>1 1</td>
<td>2.43 [2.06, 2.95]</td>
<td>2.06 [1.78, 2.39]</td>
<td>2.87 [2.30, 3.63]</td>
</tr>
<tr>
<td>$100\sigma_{PK}$, investment</td>
<td>IG</td>
<td>0.1 1</td>
<td>0.71 [0.60, 0.83]</td>
<td>1.36 [1.18, 1.56]</td>
<td>1.37 [1.19, 1.57]</td>
</tr>
<tr>
<td>$100\sigma_{\theta_{\gamma}}$, price markup</td>
<td>IG</td>
<td>0.1 1</td>
<td>0.71 [0.60, 0.83]</td>
<td>1.36 [1.18, 1.56]</td>
<td>1.37 [1.19, 1.57]</td>
</tr>
<tr>
<td>$100\sigma_{g}$, govt cons</td>
<td>IG</td>
<td>0.1 1</td>
<td>0.71 [0.60, 0.83]</td>
<td>1.36 [1.18, 1.56]</td>
<td>1.37 [1.19, 1.57]</td>
</tr>
<tr>
<td>$100\sigma_{\eta_{\theta}}$, bargaining</td>
<td>IG</td>
<td>0.1 1</td>
<td>0.71 [0.60, 0.83]</td>
<td>1.36 [1.18, 1.56]</td>
<td>1.37 [1.19, 1.57]</td>
</tr>
<tr>
<td>$100\sigma_{\tau_{\eta}}$, monetary shock</td>
<td>IG</td>
<td>0.1 1</td>
<td>0.71 [0.60, 0.83]</td>
<td>1.36 [1.18, 1.56]</td>
<td>1.37 [1.19, 1.57]</td>
</tr>
<tr>
<td>$100\sigma_{\tau_{\eta}}$, hours supply shock</td>
<td>IG</td>
<td>0.1 1</td>
<td>0.71 [0.60, 0.83]</td>
<td>1.36 [1.18, 1.56]</td>
<td>1.37 [1.19, 1.57]</td>
</tr>
</tbody>
</table>

| Distributions | N: Normal; G: Gamma; B: Beta; IG: Inverse Gamma |
ables. Posterior estimates for the inverse Frisch elasticity $\omega$ and value of the workers' bargaining power $\bar{\eta}$ are significantly different from those estimated with eight shocks. In the seven shock case, the Frisch elasticity is estimated to be in the mid-end of microeconomic estimates, which range between 0.1 and 0.6 (see Card, 1991, for a survey). By contrast, in the eight shock scenario, the value is closer to the low-end of microeconomic estimates.\(^2\) Concerning the worker's bargaining power, in the seven shock model, the posterior mean for $\bar{\eta}$ is 0.76, slightly above the range commonly used in calibrated models, 0.4 and 0.6.\(^3\) Interestingly, in the eight shock specification, $\eta$'s posterior mean drops to 0.56, in the ballpark of the estimates by Flinn (2006). All together, these results suggest that the inability of the model to account for the margins of labor adjustment is not intrinsically linked to specific parameterizations of these two labor market parameters.

The column “Preferred Model” under the heading “8 shocks” in Table 2 lists the posterior mean and 90 percentile estimates of the preferred model. First, notice that the estimate of the Frisch elasticity in the preferred model is higher than the baseline specification. Second, the posterior mean of the hours adjustment cost, $\phi_h$, is positive and equal to 6.17. To see how $\phi_h$ affects aggregate outcomes, consider the optimal condition for hours in the preferred model (i.e., with Jaimovich and Rebelo preferences):

$$\tilde{\beta}_t + \hat{h}_t - \hat{\Psi}_t + \omega \hat{h}_t + \hat{X}_t - \hat{u}_C t = \hat{\omega} + \alpha \left( \hat{u}_K t + \hat{K}_t - \hat{g}_A t - \hat{L}_t - \hat{h}_t \right) + \hat{\Delta} \hat{h}_t.$$

Solving the above expression for $\hat{h}_t$ and substituting out $\hat{\Delta} \hat{h}_t$, we obtain

$$\hat{h}_t = \frac{1}{\omega + \alpha + \phi_h} \left[ \hat{\omega} + \alpha \left( \hat{u}_K t + \hat{K}_t - \hat{g}_A t - \hat{L}_t \right) - \tilde{\beta}_t - \hat{h}_t + \hat{\Psi}_t - \hat{X}_t + \hat{u}_C t \right].$$

Thus, ultimately, a higher hours adjustment cost reduces the elasticity of hours per worker to fluctuations in the value of the marginal product of hours and the marginal utility of consumption. To get a sense of its quantitative relevance, consider the posterior-mean estimates for $\omega$ (the inverse of the Frisch elasticity) and $\phi_h$ in the preferred model: $\omega = 2.74$ and $\phi_h = 6.17$ (together with $\alpha$ calibrated at 0.3). Effectively, the hours' adjustment cost changes the inverse of the Frisch elasticity

\(^2\)Ceteris paribus, $\omega$ also affects the disutility of labor which, together with the replacement rate, determines the outside option of the worker. The latter evaluated at the posterior mean is 0.83 with seven shocks, while it is 0.69 with eight shocks. The larger outside option increases the sensitivity of the surplus and employment to aggregate shocks, other things equal.

\(^3\)A higher workers' bargaining power also increases employment's response to innovations. Ceteris paribus, a high workers' bargaining power reduces the firm's surplus, making the latter more sensitive to shocks—a mechanism in the spirit of Hagedorn and Manovskii (2008).
from 2.74 to 8.91 in the condition that determines hours per worker (since its the sum of the hours’ adjustment cost and the inverse of the Frisch elasticity that enters this condition). This value is still in the range of empirical micro-economic estimates. Importantly, introducing an hours’ adjustment cost in the model is not isomorphic to changing the Frisch elasticity altogether. The reason is that the hours’ adjustment cost does not affect the value of labor disutility that enters the worker’s surplus (relevant for wage bargaining). In this case, it is only the inverse of the Frisch elasticity that matters ($\omega$).

The posterior mean of the replacement rate $b/(wh)$ is 0.47 in the preferred model, consistent with U.S. data (OECD, 2004). In addition, the posterior interval is tighter relative to the prior. The posterior mean of the flow value of unemployment—the sum of the unemployment benefit and the real value of leisure—relative to the steady-state wage is larger and equal to 0.75, closer to the value assumed by Hall (2008). By contrast, in the baseline model with eight shocks, the replacement rate tends to be larger, with a 90 percentile interval between 0.41 and 0.69. The estimate for the bargaining power remains close to the value proposed by Flinn (2006).

The other estimated parameters are affected little across the baseline and preferred specifications with eight shocks. The shock processes in the preferred model in general have lower persistence and larger standard deviations, for instance the process for the preference shock. Overall the variability of the exogenous variables is similar across specifications.

### F Variance Decomposition

Table A.3 reports the forecast error variance decompositions at the posterior mean estimates of the preferred model for the observables, as well as for hours per worker, unemployment, and vacancies.

### G Historical Decompositions and The Margins of Labor Adjustment

Figure A.3 plots the historical decomposition of the growth rate of employment, hours per worker, and output using the posterior mean estimates of the preferred model.

The historical decompositions display the structural innovations responsible for the time-varying comovement between hours per worker and employment in U.S. recoveries. For instance, employment and hours per worker comove positively in the recoveries of the first part of the sample. Figure A.3 shows that the recoveries of 1970, 1975, and 1982 are preceded by negative investment-specific shocks, as well as negative markup shocks in 1975 and 1982 (see Appendix F for the smoothed
Table A.3: Forecast Error Variance Decompositions at Different Horizons.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Technology</th>
<th>Pref</th>
<th>Inv Specific</th>
<th>Barg Power</th>
<th>Labor Supply</th>
<th>Markup</th>
<th>Govt Spending</th>
<th>Monetary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Growth</td>
<td>0.23</td>
<td>0.09</td>
<td>0.56</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>Cons Growth</td>
<td>0.13</td>
<td>0.81</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Inv Growth</td>
<td>0.01</td>
<td>0.00</td>
<td>0.96</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>Wage Growth</td>
<td>0.03</td>
<td>0.00</td>
<td>0.00</td>
<td>0.79</td>
<td>0.00</td>
<td>0.18</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.10</td>
<td>0.04</td>
<td>0.04</td>
<td>0.01</td>
<td>0.02</td>
<td>0.75</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.08</td>
<td>0.04</td>
<td>0.07</td>
<td>0.00</td>
<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
<td>0.70</td>
</tr>
<tr>
<td>Total Hours</td>
<td>0.09</td>
<td>0.14</td>
<td>0.47</td>
<td>0.02</td>
<td>0.08</td>
<td>0.04</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>Employment</td>
<td>0.05</td>
<td>0.15</td>
<td>0.42</td>
<td>0.07</td>
<td>0.07</td>
<td>0.05</td>
<td>0.06</td>
<td>0.13</td>
</tr>
<tr>
<td>Hours per Worker</td>
<td>0.08</td>
<td>0.04</td>
<td>0.19</td>
<td>0.00</td>
<td>0.63</td>
<td>0.00</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.93</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Vacancies</td>
<td>0.05</td>
<td>0.15</td>
<td>0.42</td>
<td>0.07</td>
<td>0.07</td>
<td>0.05</td>
<td>0.06</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Figure 3. Historical decomposition for US business cycle recoveries.
shocks in the recessions and recoveries we analyze). During the recoveries, these shocks are dampened or reversed, which simultaneously boosts employment and hours per worker. By contrast, the recoveries of 1991 and 2001 feature negative comovement between employment and hours. In these episodes, the reversion of investment-specific shocks is significantly weaker. Moreover, the recoveries of 1991 and 2001 are characterized by a larger role for labor market disturbances: positive shocks to the workers’ bargaining power in 1991 and lower disutility of hours in 2001. In line with the previous discussion, both labor market shocks and the reduced importance of supply shocks break the positive comovement between the margins of labor adjustment during these recoveries.

H Sensitivity Analysis

We investigate the robustness of our results under several alternative specifications. The results of these robustness checks are summarized in Table A.4. For reference, the first two rows report the results of the baseline and preferred models, previously discussed. To understand how well the model accounts for the labor market variables, we report for each specification the shares of the variance of total hours attributed to hours per worker, employment, and their covariance. In addition, we report log marginal data densities and twice the natural logarithm of Bayes factors to provide an assessment of relative fit of models to the data, see Section 5 for more discussion. In all robustness cases, the preferred model implies a substantially greater fit. We discuss each robustness case in turn.

The preferred model includes two additional features relative to the baseline, namely JR preferences and costly hours adjustment. We consider each extension separately in the rows labeled “Model w/o JR preferences” and “Model w/o costly hours adj.” Although each feature individually improves the model’s fit relative to the baseline, the inclusion of both, the “Preferred Model” row, provides the best fit. Omitting JR preferences reduces the model’s ability to match the positive covariance between employment and hours per worker, as discussed in Section 5. In contrast, excluding the costly hours adjustment makes it more likely that the intensive margin is as volatile as the extensive margin.

Alternative Shocks

We estimate the baseline model with seven shocks when the hours supply shock is included as opposed to the bargaining power shock. Hours supply shocks can potentially improve the model’s
Table A.4: Robustness Checks from Alternative Estimated Specifications.

<table>
<thead>
<tr>
<th></th>
<th>Log Marginal Data Density vs. Preferred</th>
<th>$\beta_h$</th>
<th>$\beta_L$</th>
<th>$\beta_{av}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CES Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Models, 8 shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preferred Model</td>
<td>-1024</td>
<td>0</td>
<td>[0.13, 0.56]</td>
<td>[0.20, 0.71]</td>
</tr>
<tr>
<td>Model w/o JR preferences</td>
<td>-1041</td>
<td>34</td>
<td>[0.13, 0.57]</td>
<td>[0.24, 0.83]</td>
</tr>
<tr>
<td>Model w/o costly hours adj.</td>
<td>-1043</td>
<td>38</td>
<td>[0.14, 0.58]</td>
<td>[0.19, 0.57]</td>
</tr>
<tr>
<td>Baseline Model, right-to-manage, &amp; shock</td>
<td>-1077</td>
<td>106</td>
<td>[0.16, 0.79]</td>
<td>[0.23, 0.92]</td>
</tr>
<tr>
<td>Preferred Model, lagged costly hours adj.</td>
<td>-1043</td>
<td>38</td>
<td>[0.14, 0.58]</td>
<td>[0.19, 0.67]</td>
</tr>
<tr>
<td>Preferred Model, nominal costly hours adj.</td>
<td>-1042</td>
<td>36</td>
<td>[0.15, 0.59]</td>
<td>[0.19, 0.66]</td>
</tr>
<tr>
<td>Preferred Model, habit in leisure</td>
<td>-1046</td>
<td>44</td>
<td>[0.20, 0.69]</td>
<td>[0.14, 0.58]</td>
</tr>
<tr>
<td>Preferred Model, no consumption habit</td>
<td>-1142</td>
<td>234</td>
<td>[0.13, 0.57]</td>
<td>[0.24, 0.83]</td>
</tr>
<tr>
<td>Preferred Model, price markup estimated</td>
<td>-1025</td>
<td>2</td>
<td>[0.12, 0.54]</td>
<td>[0.21, 0.72]</td>
</tr>
<tr>
<td>Preferred Model, sep. shock, no h shock</td>
<td>-1057</td>
<td>66</td>
<td>[0.10, 0.85]</td>
<td>[0.12, 0.91]</td>
</tr>
<tr>
<td>Preferred Model, sep. shock &amp; separation data</td>
<td>-1015</td>
<td>96</td>
<td>[0.16, 0.75]</td>
<td>[0.22, 0.88]</td>
</tr>
<tr>
<td>Preferred Model, flex price &amp; wage</td>
<td>-1169</td>
<td>0</td>
<td>[0.07, 0.32]</td>
<td>[0.23, 0.62]</td>
</tr>
<tr>
<td>Baseline Model, flex price &amp; wage</td>
<td>-1256</td>
<td>174</td>
<td>[0.08, 0.71]</td>
<td>[0.24, 1.07]</td>
</tr>
<tr>
<td>Preferred Model, mix wage obs</td>
<td>-1332</td>
<td>0</td>
<td>[0.09, 0.36]</td>
<td>[0.22, 0.95]</td>
</tr>
<tr>
<td>Baseline Model, mix wage obs</td>
<td>-1380</td>
<td>96</td>
<td>[0.09, 0.72]</td>
<td>[0.26, 1.12]</td>
</tr>
<tr>
<td>Models, 7 shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 shocks, μ shock</td>
<td>-1008</td>
<td>0</td>
<td>[0.03, 0.22]</td>
<td>[0.64, 1.21]</td>
</tr>
<tr>
<td>7 shocks, h shock</td>
<td>-1076</td>
<td>136</td>
<td>[0.18, 0.60]</td>
<td>[0.10, 0.50]</td>
</tr>
<tr>
<td>7 shocks, low Frisch calibrated</td>
<td>-1016</td>
<td>16</td>
<td>[0.01, 0.07]</td>
<td>[0.77, 1.09]</td>
</tr>
<tr>
<td>7 shocks, high Frisch calibrated</td>
<td>-1025</td>
<td>34</td>
<td>[0.14, 1.10]</td>
<td>[0.46, 2.03]</td>
</tr>
<tr>
<td><strong>CPS Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Models, 8 shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preferred Model</td>
<td>-1152</td>
<td>0</td>
<td>[0.05, 0.30]</td>
<td>[0.31, 0.70]</td>
</tr>
<tr>
<td>Baseline Model</td>
<td>-1184</td>
<td>64</td>
<td>[0.11, 0.47]</td>
<td>[0.26, 0.78]</td>
</tr>
<tr>
<td><strong>SW Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Models, 8 shocks</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preferred Model</td>
<td>-989</td>
<td>0</td>
<td>[0.14, 0.62]</td>
<td>[0.18, 0.67]</td>
</tr>
<tr>
<td>Baseline Model</td>
<td>-1051</td>
<td>124</td>
<td>[0.20, 0.83]</td>
<td>[0.19, 0.80]</td>
</tr>
</tbody>
</table>

Note: Parenthesis denote 90 percent posterior intervals. Log marginal data densities calculated using Geweke’s modified harmonic mean estimator; values are comparable conditional on observables, with different sets denoted by horizontal lines.
fit with respect to labor market variables, as they directly affect the intensive labor margin. The total hours variance shares in this case are listed in row “7 shocks, $\tilde{h}$ shock” of Table A.4. For comparison, the estimates from the baseline model with seven shocks is included for reference in row “7 shocks, $\bar{\eta}$ shock.” While the hours supply shock does ensure the model matches the covariance of employment and hours per worker, it does so with a counterfactually high volatility of hours per worker, as $\beta_h$’s bands encompass higher values than $\beta_L$’s bands.

**Sensitivity to the Frisch Elasticity**

In this section, we show that the inability of the baseline model to account for the data holds independent of the specific value of the Frisch elasticity of labor supply. To illustrate the latter point, we estimated two alternative versions of the model with seven shocks, calibrating the Frisch elasticity either to be $1/7$—a much lower value than the $1/3$ estimate of the “7 shocks Baseline” specification from Table A.2, and to be one—a much higher value consistent with many macro calibrations in the literature.

Rows “7 shocks, low Frisch calibrated” and “7 shocks, high Frisch calibrated” of Table A.4 summarize the estimation results from these two versions of the model. When the Frisch elasticity is close to zero, hours’ volatility becomes counterfactually low—whereas $\beta_h$ in the data is 0.18, the posterior bands in this case range from 0.01 to 0.07. Alternatively, when the Frisch is calibrated to one, the opposite holds, and hours’ volatility become counteractually high. In both cases, the posterior estimates for the comovement between hours per worker and employment remains counterfactually low.

**Right to Manage Hours**

We now consider an alternative determination of hours, the so-called right to manage hours (henceforth RTM). Introducing RTM poses an additional empirical challenge for the baseline model, since under RTM, the hours supply shock has no direct effect on the condition that determines hours per worker and only shows up in the worker’s surplus. As a result, simply introducing RTM in the baseline model implies a much worse fit compared to our original specification. Thus, to avoid an unfair comparison, we amended the structure of the labor market shocks by considering a shock to the marginal product of hours (see the derivations below) in place of our hours supply shock.

---

4We chose $1/7$ because this is the same Frisch elasticity estimated in our Baseline model with 8 shocks (see Table A.2).
We then estimated two models: (1) the baseline model with the shock to the marginal product of hours (replacing the hours supply shock) and (2) an alternative version with RTM (replacing the neoclassical hours supply condition).

**Hours Determination**

With right-to-manage bargaining, firms maintain the right to choose the hours worked by their employees after wages have been bargained. In this case, hours per worker satisfy

\[ h_{jt} = \arg \max \left\{ S_{jt}^f \right\}, \]

where the firm-surplus is defined as in the main text:

\[
S_{jt}^f = (1 - \alpha) \varphi_t \left( \frac{rK_t}{\varphi_t \alpha} \right)^{\frac{1}{\alpha - 1}} \bar{A}_t \Delta h_{jt} + \frac{w_{jt} h_{jt}}{P_t} - \Gamma_{w, t} + E_t \beta_{t, t+1} (1 - \lambda) S_{jt+1}^f. \tag{A-8}
\]

In the symmetric equilibrium, the first-order condition implies

\[
(1 - \alpha) \varphi_t \left( \frac{rK_t}{\varphi_t \alpha} \right)^{\frac{1}{\alpha - 1}} \bar{A}_t \Delta h_{jt} = w_{jt}, \tag{A-9}
\]

where \( \Delta h_{jt} \equiv \partial h_{jt} / \partial h_{jt} \). Thus, under RTM, the hourly wage equals the value of the marginal product of hours per worker. In other words, in this alternative framework, hours supply considerations (and thus wealth effects) do not affect \( h_t \).

An implication of (A-9) is that the hours supply shock \((h_t)\) has no direct effect on the condition that determines hours per worker—\( \bar{h}_t \) only show up in the workers surplus. In turn, we have verified that this assumption implies a very poor fit of the model relative to privately-efficient bargaining.

To avoid an unfair comparison, we amended the structure of the labor market shocks by considering a shock to the marginal product of hours in place of our hours supply shock. In particular, we now assume that

\[
\tilde{h}_{jt} \equiv h_{jt}^{\tilde{\alpha}_t} \left[ 1 - \frac{\phi_h}{2} \left( \frac{h_{jt} - h_j}{h_j} \right)^2 \right],
\]

where

\[
\log \tilde{\alpha}_t = \rho_{\tilde{\alpha}} \log \tilde{\alpha}_{t-1} + \varepsilon_{\tilde{\alpha}, t}
\]
with \( \varepsilon_{\tilde{h},t} \sim N(0, \sigma^2_{\tilde{h}}) \). This assumption implies that

\[
\Delta_{\tilde{h},jt} \equiv \frac{\partial \tilde{h}_{jt}}{\partial \tilde{h}_{jt}} = \bar{\alpha}_t \tilde{h}_{jt} - \phi_h \tilde{h}_{jt} \left( \frac{h_{jt} - h_j}{h_j} \right).
\]

**Nash Bargaining**

The wage bargaining is also affected by the assumption of right to manage, since now \( \partial h_t / \partial w^n_t \neq 0 \) as implied by equation (A-9). To see this, apply the implicit function theorem. Let

\[
G(\cdot) \equiv (1 - \alpha) \varphi_t \left( \frac{r_K t}{\varphi t \alpha} \right)^{\frac{\alpha}{\alpha - 1}} \bar{A}_t \Delta_{\bar{h} t} - \frac{w^n_{jt}}{P_t}.
\]

Then

\[
\Delta_{h^n,wi} \equiv \frac{\partial h_{jt}}{\partial w^n_{jt}} = -\frac{\partial G(\cdot) / \partial w^n_{jt}}{\partial G(\cdot) / \partial h_{jt}} = \frac{1/P_t}{(1 - \alpha) \varphi_t \left( \frac{r_K t}{\varphi t \alpha} \right)^{\frac{\alpha}{\alpha - 1}} \bar{A}_t \frac{\partial \Delta_{\bar{h} t}}{\partial h_{jt}}}
\]

where

\[
\frac{\partial \Delta_{\bar{h} t}}{\partial h_{jt}} = \frac{\bar{\alpha}_t}{h_{jt}} \left( \frac{\partial \tilde{h}_{jt}}{\partial h_{jt}} \frac{1}{h_{jt}} - \tilde{h}_{jt} \right) - \phi_h \tilde{h}_{jt} - \bar{\alpha}_t \phi_h \tilde{h}_{jt} - 1 \left( \frac{h_{jt} - h_j}{h_j} \right)
\]

\[
= \bar{\alpha}_t \left( \frac{\Delta_{\bar{h} t}}{h_{jt}} - \tilde{h}_{jt} \right) - \phi_h \tilde{h}_{jt} \left[ 1 - \frac{\bar{\alpha}_t}{h_{jt}} \left( \frac{h_{jt} - h_j}{h_j} \right) \right].
\]

The firm and worker maximize the Nash product

\[
\left( S_{jt}^f \right)^{1 - \bar{\eta}_t} \left( S_{jt}^w \right)^{\bar{\eta}_t},
\]

where \( S_{jt}^f \) is defined by (A-8) and

\[
S_{jt}^w = \frac{w^n_{jt}}{P_t} h_{jt} - b \tilde{A}_t - \frac{u_{h,t}}{u_{C,t}} + \beta_{t+1} (1 - \lambda) E_t \left[ \beta_{t+1} S_{jt+1}^w \left( 1 - \frac{M_{t+1}}{U_{t+1}} \right) \right].
\]

The first-order condition with respect to \( w^n_t \) implies

\[
(1 - \bar{\eta}_t) S_{jt}^w \frac{\partial S_{jt}^f}{\partial w^n_{jt}} + \bar{\eta}_t S_{jt}^f \frac{\partial S_{jt}^w}{\partial w^n_{jt}} = 0, \quad (A-10)
\]
where

\[
\frac{\partial S^f_{jt}}{\partial w^w_{jt}} = (1 - \alpha) \varphi_t \left( \frac{\tau K_t}{\varphi_t} \right)^{\frac{\omega-1}{\omega}} A_t \Delta h^w_{jt} \Delta h^w_{jt} - \frac{h^w_{jt}}{P_t} \frac{w^w_{jt} \Delta h^w_{jt}}{P_t} - \phi^w \tilde{A}_t \left( \frac{w^w_{jt}}{w^w_{jt-1}} \pi^{t-1-1}_C \pi^{t-1}_C - \bar{g} \right) \left( \frac{w^w_{jt+1} \pi^{t+1}_C \pi^{t+1}_C - \bar{g} \pi^{t+1}_C}{(w^w_{jt})^2} \right) 
\]

and

\[
\frac{\partial S^w_{jt}}{\partial w^w_{jt}} = \frac{h^w_{jt}}{P_t} + \frac{w^w_{jt}}{P_t} \Delta h^w_{jt} - \frac{\partial u^w_{ht}t}{\partial h^w_{jt}} \Delta h^w_{jt} \frac{1}{u_C t}.
\]

Finally, as in the main paper, let

\[
\eta_{wt} = \frac{\partial S^w_{jt}}{\partial \eta^w_{jt}} = \frac{1 - \eta_t}{1 - \eta_t} \frac{\partial S^f_{jt}}{\partial \eta^w_{jt}}.
\]

The latter means that

\[
1 - \eta_{wt} = \frac{-(1 - \eta_t) \partial S^f_{jt}}{\eta \partial \eta^w_{jt} - (1 - \eta_t) \partial S^f_{jt}}.
\]

Using the above expression, the sharing rule in (A-4) can be written more compactly as

\[
(1 - \eta_{wt}) S^w_{jt} = \eta_{wt} \eta^w_{jt}.
\]

**Estimation**

Rows “Baseline Model, \(\bar{\alpha}\) shock” and “Baseline Model, right-to-manage, \(\bar{\alpha}\) shock” of Table A.4 summarize the estimation results from these two versions of the model. Comparing these rows of the table to our original baseline specification, Row “Baseline Model”, both specifications with efficient hours (rows ‘Baseline Model” and “Baseline Model, \(\bar{\alpha}\) shock”) fit the shares of the variance of total hours attributed to hours per worker, employment, and their covariance better than the right-to-manage specification (row “Baseline Model, right-to-manage, \(\bar{\alpha}\) shock”). In particular, the lack of positive comovement between employment and hours per worker is magnified under RTM. Intuitively, hours per worker equates the marginal product of an hour worked to the wage rate under RTM, implying that, with wage rigidities, hours falls when employment increases, other
things equal. This provides further support for the assumption of efficient bargaining prevailing in
the literature.

**Alternative Modeling of Costly Hours Adjustment**

In this section, we consider two alternative specifications of the costly hours adjustment: (1) an adjustment cost relative to the lagged level of hours and (2) an adjustment cost modeled as a resource cost. In addition, we assess the sensitivity of our results to modeling costly hours adjustment through habit in leisure in the household’s utility function, i.e. in terms of a cost through labor supply as opposed to through labor demand. In all cases, we show the model fit of these alternative specifications worsens relative to our original framework.

**Lagged Hours Adjustment Cost**

First, we considered a variant of the preferred model where the hours’ cost occurs in deviations of the level of hours from its previous level, rather than its steady-state level as we originally assumed.

The hours adjustment cost is now defined as

$$\phi h_{jt} \left( h_{jt} - h_{jt-1} \right)^2.$$  

As a result, the present discounted value of the stream of profits for the representative intermediate-input producer is now given by:

$$\Pi^I_{jt} \equiv E_t \left\{ \sum_{t=s}^{\infty} \beta_s \phi_s K_{js}^\alpha \left( \bar{A}_s L_{js} \tilde{h}_{js} \right)^{1-\alpha} - \frac{w^n_{ns} h_{js}}{P_s} L_{js} - \Gamma w_{js} L_{js} - \tau K_{js} - \kappa \bar{A}_s V^{1+\tau}_{js} \right\}, \tag{A-11}$$

where effective hours per worker are defined by:

$$\tilde{h}_{jt} = h_{jt} \left[ 1 - \frac{\phi h_{jt}}{2} \left( \frac{h_{jt}}{h_{jt-1}} - 1 \right)^2 \right].$$

The value of a match to the firm is

$$S^f_{jt} = \frac{\partial \Pi^I_{jt}}{\partial L_{jt}} = (1-\alpha) \phi_t \left( \frac{K_{jt}}{A_t h_{jt} L_{jt}} \right)^\alpha \bar{A}_t \tilde{h}_{jt} - \frac{w^n_{jt} h_{jt}}{P_t} - \Gamma w_{jt} + E_t \beta_{t,t+1} (1-\lambda) S^f_{jt+1}. \tag{A-12}$$

Notice that the first-order condition for $K_{jt}$ implies

$$\frac{K_{jt}}{A_t L_{jt} h_{jt}} = \left( \frac{K_{jt}}{\phi_t (A_t L_{jt} h_{jt})} \right)^{\frac{1}{\alpha-1}}.$$  

A-20
Therefore:

\[ S_{jt}^f = (1 - \alpha) \varphi_t \left( \frac{rK_t}{\varphi_t^\alpha} \right)^{\frac{\alpha}{\alpha - 1}} \tilde{A}_t \tilde{h}_{jt} - \frac{w_{jt}}{P_t} - \Gamma_{wjt} + E_t \beta_{t,t+1} (1 - \lambda) S_{jt+1}^f. \]  

(A-13)

As in the main paper, hours per worker maximize the joint surplus of the firm and worker:

\[ h_{jt} = \text{arg max} \{ S_{jt}^f + S_{jt}^w \}, \]

where \( S_{jt}^w \) denotes the worker’s surplus:

\[ S_{jt}^w = \frac{w_{jt}}{P_t} h_{jt} - bA_t - \frac{u_{hjt}}{u_{C_t}} + E_t \left[ \beta_{t,t+1} (1 - \lambda) S_{jt+1}^w \left( 1 - \frac{M_{t+1}}{U_{t+1}} \right) \right]. \]  

(A-14)

The joint surplus can then be written as:

\[ S_{jt}^f + S_{jt}^w = (1 - \alpha) \varphi_t \left( \frac{rK_t}{\varphi_t^\alpha} \right)^{\frac{\alpha}{\alpha - 1}} \tilde{A}_t \tilde{h}_{jt} - \Gamma_{wjt} - bA_t - \frac{u_{hjt}}{u_{C_t}} + E_t \beta_{t,t+1} (1 - \lambda) S_{jt+1}^w \left( 1 - \frac{M_{t+1}}{U_{t+1}} \right). \]

The derivative with respect to \( h_{jt} \) implies:

\[ \frac{\partial}{\partial h_{jt}} \left( S_{jt}^f + S_{jt}^w \right) \equiv (1 - \alpha) \varphi_t \left( \frac{rK_t}{\varphi_t^\alpha} \right)^{\frac{\alpha}{\alpha - 1}} \tilde{A}_t \frac{\partial \tilde{h}_{jt}}{\partial h_{jt}} - \frac{\partial (u_{hjt}/u_{C_t})}{\partial h_{jt}} + E_t \beta_{t,t+1} (1 - \lambda) \frac{\partial S_{jt+1}^f}{\partial h_{jt}} = 0, \]

(A-15)

where we have used the fact that

\[ \frac{\partial \Gamma_{wjt}}{\partial h_{jt}} = \frac{\partial S_{jt+1}^w}{\partial h_{jt}} = 0. \]

Finally, notice that

\[ \frac{\partial \tilde{h}_{jt}}{\partial h_{jt}} \equiv \Delta_{h_{jt}} = \frac{\tilde{h}_{jt}}{h_{jt}} - \varphi_h \left( \frac{h_{jt}}{h_{jt-1}} - 1 \right) \frac{h_{jt}}{h_{jt-1}} \]

and

\[ \frac{\partial S_{jt+1}^f}{\partial h_{jt}} \equiv \Delta_{h_{jt}} = (1 - \alpha) \varphi_{t+1} \left( \frac{rK_{t+1}}{\varphi_{t+1}^\alpha} \right)^{\frac{\alpha}{\alpha - 1}} \tilde{A}_{t+1} \frac{\partial \tilde{h}_{jt+1}}{\partial h_{jt}} \]

\[ = \varphi_h (1 - \alpha) \varphi_{t+1} \left( \frac{rK_{t+1}}{\varphi_{t+1}^\alpha} \right)^{\frac{\alpha}{\alpha - 1}} \tilde{A}_{t+1} \left( \frac{h_{jt+1}}{h_{jt}} - 1 \right) \left( \frac{h_{jt+1}}{h_{jt}} \right)^2. \]
Define \( MRS_t \equiv (\partial u_{h,jt}/\partial h_{jt})/u_{Ct} \) in equation (A-14), and let

\[
VMPH_t = (1 - \alpha) \varphi_t \left( \frac{r_{Kt}}{\varphi_t \alpha} \right)^{\alpha} \Delta_{\tilde{h},jt} + E_t \beta_{t,t+1} (1 - \lambda) \Delta_{\tilde{h},jt}^{S_{t+1}}
\]

in equation (A-15). The optimality in hours implies:

\[
VMPH_t = MRS_t.
\]

We rewrite the new equations in terms of detrended variables:

\[
\tilde{h}_{jt} = h_{jt} \left[ 1 - \frac{\phi_h}{2} \left( \frac{h_{jt}}{h_{jt-1}} - 1 \right)^2 \right]
\]

\[
MRS_t = \frac{\partial u_{h,jt}/\partial h_{jt}}{u_{Ct}}
\]

\[
VMPH_t = (1 - \alpha) \varphi_t \left( \frac{r_{Kt}}{\varphi_t \alpha} \right)^{\alpha} \Delta_{\tilde{h},jt} + E_t \beta_{t,t+1} (1 - \lambda) \Delta_{\tilde{h},jt}^{S_{t+1}} g_{A,t+1}
\]

\[
\Delta_{\tilde{h},jt}^{S_{t+1}} = \phi_h (1 - \alpha) \varphi_{t+1} \left( \frac{r_{Kt+1}}{\varphi_{t+1} \alpha} \right)^{\alpha} \left( \frac{h_{jt+1}}{h_{jt}} - 1 \right) \left( \frac{h_{jt+1}}{h_{jt}} \right)^2
\]

\[
\Delta_{\tilde{h},jt} = \frac{\tilde{h}_{jt}}{h_{jt}} - \phi_h \left( \frac{h_{jt}}{h_{jt-1}} - 1 \right) \frac{h_{jt}}{h_{jt-1}},
\]

where, as in the main paper, we do not change notation for the stationaryized variables in order to economize on notation. Next, we compute the log-linear approximation around the non-stochastic steady state:

\[
\tilde{h}_{jt} = \hat{h}_{jt}
\]

\[
VMPH_t = MRS_t
\]

\[
VMPH_t VMPH = \varphi_t \left( \frac{r_{Kt}}{\varphi_t \alpha} \right)^{\alpha} (\varphi_t - \tilde{r}_{Kt}) + (1 - \alpha) \varphi \left( \frac{r_{Kt}}{\varphi \alpha} \right)^{\alpha-1} \left( \Delta_{\tilde{h},jt} + \bar{\varphi}_t \right) + (1 - \lambda) E_t \beta g \Delta_{\tilde{h},jt}^{S_{t+1}}
\]

\[
\hat{h}_{jt} = -\phi_h \left( \hat{h}_{jt} - \hat{h}_{jt-1} \right)
\]

A-22
\[ \Delta S_{jt+1}^f = \phi_h (1 - \alpha) \varphi \left( \frac{rK_j}{\varphi \alpha} \right)^{\alpha - 1} \left( \hat{h}_{jt+1} - \hat{h}_{jt} \right). \]

Row “Preferred Model, lagged costly hours adj.” of Table A.4 provides the summary results in this case, which can be compared to row “Preferred Model” showing our original specification. As demonstrated in the table, the alternative specification has no substantial effect on the shares of the variance of total hours attributed to hours per worker, employment, and their covariance. However, our original specification implies a better fit than the lagged costly hours adjustment specification according to the Bayes factor.

**Monetary Hours Adjustment Cost**

Second, we considered a variant of the preferred model where the adjustment cost on hours is a pure monetary loss. In this case, symmetrically to the wage-adjustment cost, firms have to purchase the basket of final consumption when changing hours per worker, incurring the cost

\[ \frac{\phi_h \hat{A}_t}{2} \left( \frac{h_{jt} - h_j}{h_j} \right)^2 P_t, \]

where \( \phi_h \geq 0 \) is in units of consumption. The firm production function is now

\[ Y_{jt} = K_{jt}^\alpha (\bar{A}_t L_{jt} h_{jt})^{1-\alpha}, \]

since now \( \bar{h}_t = h_t \). The present discounted value of the stream of profits is given by:

\[ \Pi_{jt}^f = E_t \left\{ \sum_{t=s}^{\infty} \beta^{s,s+1} \left[ \varphi_s K_{js}^\alpha (\bar{A}_s L_{js} h_{js})^{1-\alpha} - \frac{w_{js} h_{js} L_{js}}{P_s} - (\Gamma_{h_{js}} + \Gamma_{w_{js}}) L_{js} - r_{K_s} K_{js} - \kappa \bar{A}_s \frac{V_{js}^{1+\tau}}{1+\tau} \right] \right\}, \]

where

\[ \Gamma_{h_{jt}} = \frac{\phi_h \hat{A}_t}{2} \left( \frac{h_{jt} - h_j}{h_j} \right)^2. \]

The value of a match to the firm is

\[ S_{jt}^f = (1 - \alpha) \varphi_t \left( \frac{rK_t}{\varphi_t \alpha} \right)^{\alpha - 1} \bar{A}_t h_{jt} - \frac{w_{jt} h_{jt}}{P_t} - (\Gamma_{h_{jt}} + \Gamma_{w_{jt}}) + E_t \beta_{t,t+1} (1 - \lambda) S_{jt+1}^f. \]
where once again we have used the first-order condition for capital. As in the main paper, hours per worker maximize the joint surplus of the firm and worker:

$$h_{jt} = \arg \max \left\{ S^f_{jt} + S^w_{jt} \right\},$$

where $S^w_{jt}$ denotes the worker’s surplus:

$$S^w_{jt} = \frac{w^n_{jt}}{P_t} h_{jt} - b\bar{A}_t - \frac{u_{h_{jt}}}{u_{C_t}} + E_t \left[ \beta_{t,t+1} (1 - \lambda) S^w_{jt+1} \left( 1 - \frac{M_{t+1}}{U_{t+1}} \right) \right].$$

Therefore the joint surplus can be written as:

$$S^f_{jt} + S^w_{jt} = (1 - \alpha) \varphi_t \left( \frac{r K_t}{\varphi_t \alpha} \right)^{\frac{\alpha}{\alpha - 1}} \bar{A}_t - \left( \Gamma_{h_{jt}} + \Gamma_{w_{jt}} \right) b\bar{A}_t$$

$$- \frac{u_{h_{jt}}}{u_{C_t}} + E_t \beta_{t,t+1} (1 - \lambda) S^f_{jt+1} + E_t \left[ \beta_{t,t+1} (1 - \lambda) S^w_{jt+1} \left( 1 - \frac{M_{t+1}}{U_{t+1}} \right) \right].$$

The derivative with respect to $h_{jt}$ implies:

$$\frac{\partial \left( S^f_{jt} + S^w_{jt} \right)}{\partial h_{jt}} = (1 - \alpha) \varphi_t \left( \frac{r K_t}{\varphi_t \alpha} \right)^{\frac{\alpha}{\alpha - 1}} \bar{A}_t - \frac{u_{h_{jt}}}{u_{C_t}} \left( h_{jt} - h_j \right) - \frac{\partial u_{h_{jt}} / \partial h_{jt}}{u_{C_t}} = 0,$$

where we have used the fact that

$$\frac{\partial \Gamma_{w_{jt}}}{\partial h_{jt}} = \frac{\partial S^w_{jt+1}}{\partial h_{jt}} = \frac{\partial S^f_{jt+1}}{\partial h_{jt}} = 0.$$

Let $MRS_t \equiv \left( \frac{\partial u_{h_{jt}} / \partial h_{jt}}{u_{C_t}} \right).$ We then finally obtain:

$$VMPH_t = MRS_t,$$

where

$$VMPH_t \equiv (1 - \alpha) \varphi_t \left( \frac{r K_t}{\varphi_t \alpha} \right)^{\frac{\alpha}{\alpha - 1}} \bar{A}_t - \phi_h \bar{A}_t \left( \frac{h_{jt} - h_j}{h_j} \right).$$
Combining the household’s and government’s budget constraints now yields the following aggregate resource constraint:

\[
Y^C_t \left[ 1 - \frac{\nu}{2} \left( \pi_{Ct}^{\nu_C} \pi_{Ct-1}^{\nu_C} - 1 \right)^2 \right] = (A-16)
\]

\[
C_t + I_{Kt} + \kappa_t \hat{A}_t + G_t + \frac{\phi^w \hat{A}_t}{2} \left( \pi_{wt} \pi_{Ct}^{\nu_C} - \bar{\eta} \right)^2 L_t + \frac{\phi_h \hat{A}_t}{2} \left( \frac{h_{jt} - h_j}{h_j} \right)^2 L_t.
\]

Intuitively, total output produced by firms must now be equal to the sum of market consumption, investment in physical capital, the costs associated to job creation, the purchase of goods from the government, and the real cost of changing prices, wages, and hours. In a first-order approximation to the model policy functions, the extra term in the resource constraint from the cost of adjusting hours disappears, since the cost is zero in steady state. For this reason we omit this equation below.

We rewrite the new equations in terms of detrended variables:

\[
MRS_t = \frac{\partial u_{hjt}/\partial h_{jt}}{u_{Ct}}
\]

\[
VMPH_t = (1 - \alpha) \varphi_t \left( \frac{r_{Kt}}{\varphi_t \alpha} \right) \frac{\alpha}{\alpha-1} - \phi_h \left( \frac{h_{jt} - h_j}{h_j} \right)
\]

where, as in the main paper, we do not change notation for the stationarized variables in order to economize on notation. Next, we compute the log-linear approximation around the non-stochastic steady state:

\[
\hat{VMPH}_t = \hat{MRS}_t
\]

\[
\hat{VMPH}_t \hat{VMPH} = \varphi \alpha \frac{1}{\alpha-1} \left( \frac{r_{K}}{\varphi} \right) \frac{\alpha}{\alpha-1} (\hat{r}_t - \hat{r}_{Kt}) + (1 - \alpha) \varphi \left( \frac{r_{K}}{\varphi \alpha} \right) \frac{\alpha}{\alpha-1} \hat{r}_t - \phi_h \hat{h}_{jt}.
\]

Row “Preferred Model, nominal costly hours adj.” of Table A.4 shows that this alternative specification also has no substantial effect on the shares of the variance of total hours attributed to hours per worker, employment, and their covariance, relative to our original specification. However, our original specification implies a better fit according to the Bayes factor.

**Habit in Leisure**

Finally, we considered an alternative where the cost to adjustment in hours instead falls on households by modeling habit in leisure in the household’s utility function. For this reason, in this version of the model, \( \phi_h = 0 \).
Household preferences are now given by
\[
W_t \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} \beta_s \left[ \log \left( C_t - h_C C_{t-1} - \bar{h}_t X_t \int_{0}^{L_t} \frac{(h_{jt} - h_h h_{t-1})^{1+\omega}}{1+\omega} dj \right) \right],
\]
where \( \gamma \in (0, 1] \), \( X_t = (C_t - h_C C_{t-1})^\gamma X_{t-1}^{1-\gamma} \), and
\[
h_t = \int_{0}^{L_t} h_{jt} dj.
\]
This alternative utility function affects (i) the marginal rate of substitution between consumption and leisure and (ii) the worker’s outside option at the wage bargaining stage. As a result, both the optimality condition for hours per worker and the bargained wage change.

Privately efficient hours continue to equate the marginal rate of substitution between consumption and leisure to the value of the marginal product of an hour worked. However, in this case the marginal rate of substitution is given by
\[
-\frac{\partial u_{h_{jt}}}{\partial h_{jt}} \frac{u_{C_t}}{u_{C_t}} = -\Psi_t^{-1} \beta_t \bar{h}_t X_t (h_{jt} - h_h h_{t-1})^{1+\omega},
\]
where
\[
\Psi_t \equiv C_t - h_C C_{t-1} - \bar{h}_t X_t \int_{0}^{L_t} \frac{(h_{jt} - h_h h_{t-1})^{1+\omega}}{1+\omega} dj.
\]
The opportunity cost of giving up leisure for an employed worker (expressed in units of consumption) is now given by:
\[
ocl_t \equiv \Psi_t^{-1} \beta_t \bar{h}_t X_t (h_{t} - h_h h_{t-1})^{1+\omega} / (1 + \omega) u_{C_t}.
\]
In log-linear terms the equations are as follows:
\[
\overline{MRS}_t = -\Psi_t + \tilde{\beta}_t + \hat{h}_t + \hat{X}_t + \frac{\omega}{(1 - h_h)} \left( \hat{h}_t - h_h \hat{h}_{t-1} \right),
\]
\[
\overline{ocl}_t = -\Psi_t + \tilde{\beta}_t + \hat{h}_t + \hat{X}_t + \frac{1 + \omega}{(1 - h_h)} \left( \hat{h}_t - h_h \hat{h}_{t-1} \right) - \hat{u}_{C,t}.
\]
Row “Preferred Model, habit in leisure” of Table A.4 provides the summary results from this specification, which can be compared to row “Preferred Model” showing our original specification. Habit in leisure has somewhat-similar shares of the variance of total hours attributed to hours per worker, employment, and their covariance; notably though, the variance share attributed to hours
increases to the point that the share in the data is outside the posterior distribution. This helps explain why the model has a substantially worse fit, according to the Bayes factor, relative to our original specification.

**Sensitivity to Steady-State Price Markups**

To understand how sensitive our estimates are to the calibration of the steady-state price markup, we re-estimated the preferred model with the addition of the steady-state price markup as a parameter to be estimated. In this case, we adopted a prior with a normal distribution centered at 1.25 and standard deviation of 0.07. Thus, the three standard deviation range of the prior for the price markup ranges from 1.04 to 1.46. In contrast, the estimated 90% bands for the posterior distribution of the price markup are from 1.11 to 1.25, and the posterior mean is 1.18.

Row “Preferred Model, price markup estimated” of Table A.4 provides our summary results from this case, which can be compared to row “Preferred Model” showing our original specification. As demonstrated in the table, the alternative specification has virtually identical implied shares of the variance of total hours attributed to hours per worker, employment, and their covariance. Moreover, the two specifications imply an indistinguishable fit according to the Bayes factor.

**The Role of Habit in Consumption**

To assess the sensitivity of our results to habit in consumption, we estimated the preferred model with the consumption habit parameter calibrated to zero. Row “Preferred Model, no consumption habit” of Table A.4 summarizes the results in this case. Without habit, the preferred model is more likely to imply a negative covariance between the intensive and extensive margins.

More importantly, the model without consumption habit has trouble reproducing the persistence of consumption’s correlation with itself and other macro variables, such as total hours and hours per worker. This can be seen in Figure A.4, which displays correlograms from the preferred model without consumption habit (black dashed lines) relative to the preferred model estimated with consumption habit (red dotted-dashed lines). Notably, several data moments lie outside the bands for the model estimated without consumption habit.

**The Role of Nominal Rigidities**

To assess the importance of nominal rigidities for our results, we estimated a real version of the preferred model and baseline models. In this case, we removed inflation and the nominal interest
Figure 4. Correlograms from the data (solid lines) and 90 percent posterior intervals from 1) the preferred model estimated without consumption habits (black dashed lines) and 2) the preferred model (red dotted-dashed lines).
rate from the observables used for estimation. This experiment allowed us to (i) assess the fit of
the preferred model in the absence of nominal rigidities; and (ii) assess whether the inability
of the baseline model to reproduce the key comovements of the margins of labor is sensitive to the
inclusion of nominal rigidities.

Rows “Baseline Model, flex price & wage” and “Preferred Model, flex price & wage” of Table
A.4 report our summarized results. Since the Bayes factor can only be employed to compare
models that have the same sets of observables, we cannot directly compare the fit of the preferred
model with flexible prices and wages (which abstracts from two observables) to our estimated
version. However, we can compare the estimated real versions of the baseline and preferred models,
which contain the same six observables. Rows “Baseline Model, flex price & wage” and “Preferred
Model, flex price & wage” show that even a real version of the baseline model implies the same
counterfactual comovements in the labor margins, while the real version of the preferred model
substantially improves the model fit relative to the baseline, as seen from the implied Bayes factor.

In addition, the absence of nominal rigidities fundamentally alters the variance decomposition
of the model. In particular, labor-market shocks become substantially more prominent in driving
fluctuations in the margins of labor. For instance, hours supply shocks and shocks to workers’
bargaining power in the preferred model now account for 91 percent of employment variation after
10 periods and 85 percent on impact. In contrast, in the model with nominal rigidities, these shocks
account for only 14 (15) percent on impact (after 10 periods). We view these results as indirectly
confirming that our results about the features needed to capture labor and macro comovements do
not hinge on the specific source of employment volatility—i.e., whether or not wage rigidities are
the key driver of extensive-margin dynamics.

**Incorporating Separation Shocks and Data on Separation Rate**

In this section, we consider the sensitivity of our results to including exogenous fluctuations in the
separation rate in our preferred model. First, row “Preferred Model, sep. shock, no \( \bar{h} \) shock” in
Table A.4 provides the results from the preferred model when the hours supply shock is replaced
with a shock to the separation rate. That is, we assume the separation probability \( \lambda \) is no longer
constant, but follows an exogenous AR(1) process:

\[
\bar{\lambda}_t = \rho_\lambda \bar{\lambda}_{t-1} + (1 - \rho_\lambda) \lambda + \epsilon_\lambda_t,
\]

Incorporating Separation Shocks and Data on Separation Rate
where with $\bar{\epsilon}_t \sim iid N\left(0, \sigma^2\right)$.

Our original specification with the hours supply shock (row “Preferred Model”) implies a substantially greater fit than the separation shock specification (row “Preferred Model, sep. shock, no $\bar{h}$ shock”). In this case, the Bayes factor for the separation shock version is 66, well above the threshold of Kass and Raftery (1995) for evidence in favor of our original model with the hours supply shock. Moreover, the separation shock substantially increases the volatility of the variance of total hours attributed to hours per worker and employment, as well as admitting a much larger range of negative covariances between hours per worker and employment. These results reflect the facts that the shock does not directly affect the intensive margin, and no data related to separation is included in the estimation. In fact, when we compare the smoothed estimate of the estimated separation shock to a measure of the separation rate (from Shimer’s quarterly employment exit probability, available at https://sites.google.com/site/robertshimer/research/flows), we find the two series have very low correlation.

Because of this mismatch, we have also conducted a second experiment and verified that adding to our original estimation data on the separation margin together with the separation shock does not alter our conclusions or messages. To do so, we constructed an observable series for the separation rate from Shimer’s quarterly employment exit probability mentioned above. This series is linked one-for-one to the exogenous separation probability shock $\bar{\lambda}_t$ in the measurement equation of our state-space model. We re-estimated both the baseline and preferred models with the addition of this separation series, as well as the separation shock. Rows “Baseline Model, sep. shock & separation data” and “Preferred Model, sep. shock & separation data” of Table A.4 report our summarized results. As was the case with our original estimation, the preferred model is still substantially preferred to the baseline, as documented by the Bayes factor comparison. Figure A.5 shows the differences in fit can be traced to the same issues with our original estimation, namely the correlograms between hours per worker and consumption, hours per worker and output, and hours per worker and employment on impact. Moreover, the shares of the variance of total hours attributed to hours per worker, employment, and their covariance for the two specifications are quite similar to our original estimates for the two models. All in all, while the employment exit probability data helps inform the separation shock, the shock itself cannot account for the relative movements of the intensive and extensive labor margins. The hours supply shock, along with Jaimovich-Rebelo preferences and costly hours adjustment, accounts for the cyclical properties of the labor margins.
Figure 5. Correlograms from the data (solid lines) and 90 percent posterior intervals from 1) the benchmark model augmented with exogenous separation shocks and separation data (black dashed lines) and 2) the preferred model augmented with exogenous separation shocks and separation data (red dotted-dashed lines).

Wage Data

We document the robustness of our results to the wage observable. Using U.S. micro data, Haefke et al. (2013) document that the wages of newly hired workers, unlike wages in ongoing relationships, are volatile and procyclical. In addition, our baseline wage observable is not restricted to earnings, as it includes employer contributions to employee-benefits (Justiniano et al., 2013). We address these issues as follows. We estimate a version of the preferred model in which three measures of the wage are simultaneously included in the observables. This strategy has been recently used by several papers in the estimation literature (see for instance Boivin and Giannoni (2006), Gali et al. (2011), and Justiniano et al. (2013)). The first is the measure described in Section 4 of the main text, which is the BLS’ hourly compensation for the nonfarm business sector. The second measure is the BLS’ average hourly earnings of production and nonsupervisory employees. The third measure is the quality adjusted wage series of Haefke et al. (2013), which adjusts for individual-level characteristics. We assume that each series represents an imperfect measure of the
model wage according to:

\[
\begin{bmatrix}
\text{Comp Wage}_t \\
\text{Earn Wage}_t \\
\text{Quality Wage}_t
\end{bmatrix}
= \begin{bmatrix}
\Gamma_1 \\
\Gamma_2 \\
\Gamma_3
\end{bmatrix}
\begin{bmatrix}
\hat{w}_t - \hat{w}_{t-1} + \hat{g}_{At} \\
\hat{e}_{1t} \\
\hat{e}_{2t} \\
\hat{e}_{3t}
\end{bmatrix}
\]

where \(e_{it}\) for \(i = 1, 2, 3\) denote iid observation errors.\(^5\) Rows “Preferred Model, mix wage obs” and “Baseline Model, mix wage obs” of Table A.4 display the total hours variance shares in this case. Again, the preferred model has a better fit, with bands well encompassing the data.

**Alternative Labor Market Variables and Subsample Analysis**

We check whether our results are sensitive to the labor market measures used for the estimation. We estimate the model using CPS labor market variables, as in Ramey (2012).\(^6\) In this case, neither total hours nor employment are linearly detrended as it is less obvious the series exhibit a deterministic trend; the two variables are demeaned. Parameter estimates in this case are comparable to those in Table A.2. Bayes factors suggest strong preference for the preferred model as well. As shown in Table A.4, the posterior bands for the model’s \(\beta\)s well-encompass their data counterparts. In addition, these results are robust to using the Smets and Wouters (2007) labor market observables for estimation, which are commonly employed in the DSGE estimation literature, as evidenced by the last rows of Table A.4.

Finally, our analysis of U.S. recoveries is robust to sub-sample estimation conditional on our observables. This experiment allows us to address how structural change in parameter estimates (in particular, those directly affecting labor market dynamics) contributes to the dynamics of hours and employment in post-war U.S. data (the results are available upon request). As is common practice in the literature, we split our original sample at the start of the so-called Great Moderation, estimating from 1965:Q1 to 1983:Q4 and 1984:Q1 to 2007:Q4.

---

\(^5\)The priors for the \(\Gamma\)’s are normal distributions centered at 1 with a standard deviation of 0.5. The priors for the standard deviations of the wage observation errors are inverse gamma distributions with mean of 0.1 and standard deviation of 1. Specifically, we use the median real wage of new hires corrected for fluctuations in all observable worker characteristics from Haefke et al. (2013). This series is not available for the full sample period, but the Kalman filter handles missing observations.

\(^6\)See Appendix A for a description of the alternative labor market data.
Table A.5: Model Equations, Stationary Deviations from Trend

(1) \( L_t = (1 - \lambda) L_{t-1} + M_t \)

(2) \( \frac{\beta \delta h_t \bar{C}^\alpha}{\bar{u} C_t} = (1 - \alpha) \varphi_t \left( \frac{W_t K_t}{\bar{u} A_t L_t h_t} \right) \)

(3) \( \bar{K}_{t+1} = (1 - \delta K_t) \frac{\bar{K}_t}{\bar{A}_t} + \bar{p}^K I_{Kt} \left[ 1 - \frac{\nu}{2} \left( \bar{g}_{A_t} \frac{I_{Kt}}{\bar{K}_{t-1}} - \bar{g}_A \right)^2 \right] \)

(4) \( r_{Kt} = \zeta_{Kt} \left[ \delta_{K1} + \delta_{K2} (u_{Kt} - 1) \right] \)

(5) \( \zeta_{Kt} = \beta E_t \left( \frac{\bar{u} C_t + 1}{u_{Ct}} \right) \left[ \frac{r_{Kt+1} + u_{Kt+1} + (1 - \delta_{Kt+1}) \zeta_{Kt+1}}{1} \right] \)

(6) \( 1 = \zeta_{Kt} \bar{p}^K \left[ 1 - \frac{\nu}{2} \left( \bar{g}_{A_t} \frac{I_{Kt}}{\bar{K}_{t-1}} - \bar{g}_A \right)^2 \right] - \nu \left( \bar{g}_{A_t} \frac{I_{Kt}}{\bar{K}_{t-1}} - \bar{g}_A \right) \left( \bar{g}_{A_t} \frac{I_{Kt}}{\bar{K}_{t-1}} - \bar{g}_A \right) \left( \frac{I_{Kt}}{\bar{K}_{t-1}} \right)^2 \)

(7) \( M_t = \bar{\chi}_t U_{\bar{V} t}^{1-x} \)

(8) \( \kappa_{\bar{V} t} = S_t \)

(9) \( \eta_w t S_t = (1 - \eta_w t) S_t \)

(10) \( \pi_{w t} = \bar{g}_{A_t} \frac{u_{Ct}}{u_{Ct+1}} \pi_{Ct} \)

(11) \( 1 = \beta_t E_t \left[ \frac{\bar{u} C_t + 1}{u_{Ct} \bar{g}_{A_t+1} \pi_{Ct+1}} \right] \)

(12) \( \frac{\bar{\xi}_t}{\varphi_t} = (1 - \nu) \bar{p}^\alpha \left[ \left( \frac{\bar{u} C_t}{\bar{u} C_t + 1} \right) \beta E_t \left( \frac{\bar{u} C_t}{\bar{u} C_t + 1} \right) \right] \)

(13) \( 1 = \frac{1}{\varphi_t} \left( \frac{\varphi_t}{\varphi_t} \right) \)

(14) \( \left( \frac{\bar{u} C_t K_t}{\bar{u} C_t L_t h_t} \right)^\alpha \left( L_t h_t \right)^{1-\alpha} \left[ 1 - \frac{\nu}{2} \left( \pi_{Ct} \pi_{Ct-1} \pi_{Ct-1} - 1 \right)^2 \right] = C_t + I_{Kt} + \kappa_t V_t + G_t \)

(15) \( r_{Kt} = \varphi_t \alpha \left( \frac{w_{Kt} K_t}{u_{Kt} L_t h_t} \right) \)

D.1, I_t = \beta_t E_t \left( \frac{1}{u_{Ct} \bar{g}_{A_t+1} \pi_{Ct+1}} \right) \)

D.2, \( U_t = 1 - (1 - \lambda) L_{t-1} \)

D.3, \( \bar{\Xi}_t = 1 - \frac{\varphi_t}{\varphi_t} \left( \pi_{Ct} \pi_{Ct-1} \pi_{Ct-1} - 1 \right)^2 + \frac{\varphi_t}{\varphi_t} \left( \pi_{Ct} \pi_{Ct-1} \pi_{Ct-1} - 1 \right) - \frac{\pi_{Ct} \pi_{Ct-1} \pi_{Ct-1} - 1}{\varphi_t} \left( \pi_{Ct} \pi_{Ct-1} \pi_{Ct-1} - 1 \right) \)

D.4, \( S_t = (1 - \alpha) \varphi_t \left( \frac{w_{Kt} K_t}{u_{Kt} L_t h_t} \right) \left( h_t - w_t h_t - \frac{\varphi_t}{2} \left( \pi_{w t} \pi_{Ct-1} \pi_{Ct-1} - 1 - \bar{g}_A \right)^2 \right) + (1 - \lambda) \beta E_t \left( \frac{u_{Ct+1} S_{t+1}}{u_{Ct} \bar{g}_{A_t+1} \pi_{Ct+1}} \right) \)

D.5, \( S_{t} = w_t h_t - b - \frac{\beta_t h_t}{\varphi_t} \left( \pi_{w t} \pi_{Ct-1} \pi_{Ct-1} - 1 - \bar{g}_A \right)^2 + (1 - \lambda) \beta E_t \left( \frac{u_{Ct+1} S_{t+1}}{u_{Ct} \bar{g}_{A_t+1} \pi_{Ct+1}} \right) \)

D.6, \( u_{Ct} = \beta_t \left( \frac{1}{(C_t - h_t)^2} \right) - \beta C E_t \left( \frac{1}{1 - h_t (C_t - h_t)^2} \right) \)

D.7, \( G_t = \frac{M_t}{\bar{V} t} \)

D.8, \( \delta_{Kt} = \delta_{K0} + \delta_{K1} (u_{Kt} - 1) + (\delta_{K2} / 2) (u_{Kt} - 1)^2 \)

D.9, \( \kappa_t = \kappa V_{\bar{V} t}^\alpha / (1 + \tau) \)

D.10, \( \varphi = \varphi - \varepsilon_q t = \varepsilon_g t = 0. \)

Note: \( C_t \) and \( I_{Kt} \) in equation (D.1) are consumption and investment observed when \( \varphi = \varphi = \varepsilon_q t = \varepsilon_g t = 0. \)
Variable without a time subscript denotes steady-state values.
\[
\frac{\Psi_t}{u_{C_t}} = (1 - \alpha) \varphi_t \left( \frac{K_t}{g_{At}h_t} \right)^a \Delta_{ht}.
\]

\[
\left( \frac{u_{C_t}K_t}{g_{At}} \right)^\alpha \left( L_t\tilde{h}_t \right)^{1-a} \left[ 1 - \frac{w}{\tilde{h}_t} \left( \pi_{Ct}^{1-w} \pi_{Ct-1}^{-w} - \tilde{h}_t \right) \right] = C_t + I_{Kt} + \kappa_tV_t + G_t
\]

\[
r_{Kt} = \varphi_t \alpha \left( \frac{u_{C_t}K_t}{g_{At}h_t} \right)^{a-1}
\]

\[
S^f_t = (1 - \alpha) \varphi_t \left( \frac{u_{C_t}K_t}{g_{At}h_t} \right)^\alpha \tilde{h}_t - w_t h_t - \frac{g}{\tilde{h}_t} \left( \pi_{Ct}^{1-w} \pi_{Ct-1}^{-w} - g_A \right)^2 + (1 - \lambda) \beta E_t \left( \frac{u_{Ct+1}S^w_{t+1}}{u_{C_t}} \right)
\]

\[
S^w_t = w_t h_t - b - \frac{\Psi_{t-1}^h}{(1+\omega)u_{C_t}} + (1 - \lambda) \beta E_t \left( \frac{u_{Ct+1}S^w_{t+1}}{u_{C_t}} \right)
\]

\[
u_{Ct} = \left[ -\tilde{h}_t^{\alpha-1} + \gamma \mu_t \left( C_t - h_t \frac{C_{t-1}}{g_{At}} \right)^{\gamma-1} \frac{X_t^{1-\gamma} \gamma \gamma^{-1} - \beta h_C E_t \left[ \tilde{h}_{t+1} (\Psi_{t+1} g_{At+1})^{-1} \right]}{\gamma h_C E_t \left[ \frac{\Psi_{t+1} g_{At+1}}{g_{At+1}} \right]} \right]
\]

\[
\mu_t = \frac{\Psi^{t-1}}{u_{Ct}} \left( L_t \tilde{h}_t \right)^{1+w} + (1 - \gamma) \beta E_t \left[ \frac{\Psi_{t+1} g_{At+1}}{g_{At+1}} \right] \left( C_t - h_t C_t \right)^{\gamma} \tilde{X}_t^{1-\gamma}
\]

\[
\Psi_t = C_t - h_t \frac{C_{t-1}}{g_{At}} - \frac{h_t L_t h_t^{1+w} X_t}{1+w}
\]

\[
X_t = \left( C_t - h_t \frac{C_{t-1}}{g_{At}} \right)^{\gamma} \frac{X_t^{1-\gamma}}{1-\gamma}
\]

\[
\tilde{h}_t = h_t \left[ 1 - \frac{\phi}{2} (h_t - h)^2 \right]
\]

\[
\Delta_{ht} = \frac{h_t}{h_t - \phi h_t (h_t - h)}
\]

Note: Other equations are unchanged relative to Table A.5.
TABLE A.7: LOG-LINEARIZED MODEL EQUATIONS

(1) \[ LL_t = L(1 - \lambda)L_{t-1} + MM_t \]

(2) \[ \hat{\beta} + \hat{h} + \omega \hat{h} - \hat{\alpha} = \hat{\beta} + \alpha \left( \hat{u}_{Kt} + \hat{K}_t - \hat{g}_{At} - \hat{L}_t - \hat{h}_t \right) \]

(3) \[ \hat{K}_{t+1} = \frac{(1-\delta_K)}{\delta_A} \left( \hat{K}_t - \hat{g}_{At} \right) + \left( 1 - \frac{1-\delta_K}{\delta_A} \right) \left[ \hat{I}_{t} + \hat{I}_{Kt} \right] \]

(4) \[ \hat{r}_{Kt} = \hat{\alpha}_{Kt} + \frac{1}{\hat{\alpha}_{Kt}} \hat{\beta}_{Kt} \]

(5) \[ \hat{\alpha}_{Kt} = E_t \hat{u}_{Ct+1} - \hat{u}_{Ct} - E_t \hat{g}_{At+1} + \frac{\beta}{\delta_A} \hat{r}_K \left( E_t \hat{r}_{Kt+1} + E_t \hat{u}_{Kt+1} + \frac{\beta}{\delta_A} (1 - \delta_K) E_t \hat{\alpha}_{Kt+1} - \frac{\beta}{\delta_A} \delta_K E_t \hat{u}_{Kt+1} \right) \]

(6) \[ (1 + \beta) \hat{I}_{kt} - \frac{1}{\delta_A} \left( \hat{\alpha}_{Kt} + \hat{\beta}_{Kt} \right) - \beta E_t \hat{I}_{Kt+1} + \hat{g}_{At} - \beta E_t \hat{g}_{At+1} = \hat{I}_{Kt-1} \]

(7) \[ \hat{M}_t = \hat{\epsilon} + \epsilon \hat{U}_t + (1 - \epsilon) \hat{V}_t \]

(8) \[ \hat{\tau} \hat{V}_t - \hat{q}_t = \hat{S}_t^\text{f} \]

(9) \[ \hat{q}_t + \hat{\beta} \hat{t} = \hat{S}_t^\text{w} \]

(10) \[ \hat{A}_t = \hat{g}_{At} + \hat{w}_t - \hat{w}_{t-1} - \hat{\pi}_{At} \]

(11) \[ \hat{h}_t + E_t \hat{u}_{Ct+1} - \hat{u}_{Ct} - E_t \hat{g}_{At+1} - E_t \hat{\alpha}_{Ct+1} = 0 \]

(12) \[ \hat{h}_t = \hat{g}_t - \hat{u}_{Ct+1} - (1 - \hat{g}_t) \hat{\phi}_{Ct+1} \]

(13) \[ 0 = -\frac{1}{\delta_A} \hat{\alpha}_{At} + \hat{\alpha}_{At} + \hat{\beta}_{At} \]

(14) \[ \alpha \left( \hat{u}_{Kt} + \hat{K}_{t-1} - \hat{g}_{At} \right) + (1 - \alpha) \left( \hat{L}_t + \hat{h}_t \right) = \hat{C}_t + \frac{\hat{h}_t}{\hat{\epsilon}^2} \left( \hat{K}_t + \hat{I}_{Kt} - \hat{g}_{At} - \hat{L}_t - \hat{h}_t \right) \]

(15) \[ \hat{r}_{Kt} = \hat{\alpha} + (\alpha - 1) \left( \hat{u}_{Kt} + \hat{K}_t - \hat{g}_{At} - \hat{L}_t - \hat{h}_t \right) \]

(D.1) \[ \hat{Y}_t = C \left( \hat{C}_t - \hat{\pi}_C \right) + I_K \left( \hat{I}_{Kt} - \hat{I}_{Kt} \right) + G \hat{G}_t \]

(D.2) \[ \hat{U}_t = -(1 - \lambda) \hat{L}_{t-1} \]

(D.3) \[ \hat{z}_t = \frac{1}{\lambda} \left[ \left( 1 - \alpha \right) \hat{\phi} \left( \hat{S}_t^f - \hat{S}_t^w \right) - \hat{\phi} \beta \left( \hat{E}_t \hat{\pi}_{pt+1} - \hat{t}_p \hat{\pi}_{pt} \right) \right] \]

(D.4) \[ \hat{S}_t^f \hat{S}_t^w = \left[ \frac{1}{\alpha} \hat{\phi} \left( \frac{\hat{u}_{Kt} + \hat{K}_t - \hat{g}_{At} - \hat{L}_t} \right) + (1 - \alpha) \hat{h}_t \right] \]

(D.5) \[ \hat{S}_w \hat{S}_w = \left[ \hat{wh}(\hat{u}_t + \hat{h}_t) - \frac{1}{\alpha} \hat{\phi} \left( \hat{u}_{Kt} + \hat{K}_t - \hat{g}_{At} - \hat{L}_t \right) + (1 - \alpha) \hat{h}_t \right] \]

(D.6) \[ \hat{u}_{Ct} = \left[ \frac{g_A}{\delta_A - \beta h} \left( \frac{g_A}{\delta_A - \beta h} \right) \hat{E}_t \hat{C}_{t+1} - \frac{g_A}{\delta_A - \beta h} \right] \]

(D.7) \[ \hat{q}_t = \hat{M}_t - \hat{V}_t \]

(D.8) \[ \hat{\xi}_{Kt} = \hat{\xi}_{Kt} + \hat{\beta}_{Kt} \]

(D.9) \[ \hat{K}_t = \hat{V}_t \]

(D.10) \[ \hat{\eta}_t = \hat{\eta}_t - \frac{1}{\beta} \left[ \frac{\hat{h}_t}{\alpha} \hat{h}_t - \hat{h}_t \hat{h}_t + (1 - \hat{h}_t) \right] \]

Note: \( C_t \) and \( I_{Kt} \) in equation (D.1) are consumption and investment observed when \( \phi^w = \phi^f = \epsilon = \epsilon_{At} = 0 \). Variables without a time subscript denote steady-state values; \( \varsigma = \delta_{K2}/\delta_{K1} \).
\[ \begin{align*}
\text{(2') } & \quad \tilde{\beta}_t + \tilde{h}_t - \Psi_t + \omega \tilde{h}_t + \tilde{X}_t - \tilde{u}_{Ct} = \tilde{\varphi}_t + \alpha \left( \tilde{u}_{Kt} + \tilde{K}_t - \tilde{g}_{At} - \tilde{L}_t - \tilde{h}_t \right) + \tilde{\Delta}_{ht} \\
\text{(14') } & \quad \alpha(\tilde{u}_{Kt} + \tilde{K}_{t-1} - \tilde{g}_{At}) + (1 - \alpha)(\tilde{L}_t + \tilde{h}_t) = \tilde{C}_t + \frac{\bar{k}}{\lambda} \tilde{I}_{Kt} + \frac{\kappa^{y_{t+1}}}{\lambda} \left( \tilde{K}_t + \tilde{V}_t \right) + \frac{\bar{C}}{\lambda} \tilde{C}_t \\
\text{(15') } & \quad \tilde{r}_K = \tilde{\varphi}_t + (\alpha - 1) \left( \tilde{u}_{Kt} + \tilde{K}_t - \tilde{g}_{At} - \tilde{L}_t - \tilde{h}_t \right)
\end{align*} \]

\[ \begin{align*}
\text{(D.4') } & \quad S_t^l S_t^l = \left[ (1 - \alpha) \varphi \left( \frac{\omega h_{\lambda L} \hat{K}}{1 + \omega} \right)^{\alpha} \left( \hat{a}_{Kt} + \hat{K}_t - \hat{g}_{At} - \hat{L}_t \right) + (1 - \alpha) \hat{h}_t \right] \\
& \quad - \omega h(\tilde{w}_t + \hat{h}_t) + \beta(1 - \lambda) S_t^l \left[ E_t \tilde{u}_{Ct+1} - \tilde{u}_{Ct} + E_t \tilde{S}_{t+1}^l \right]
\end{align*} \]

\[ \begin{align*}
\text{(D.5') } & \quad S_t^w \tilde{S}_t^w = \left[ \omega h(\tilde{w}_t + \hat{h}_t) - \psi^{-1} \beta h^{1 + \omega} X_t \left( \frac{\beta}{1 + \omega} \right) \tilde{h}_t - \hat{\Psi}_t + \hat{h}_t + (1 + \omega) \hat{h}_t + \tilde{X}_t - \tilde{u}_{Ct} \right] \\
& \quad + (1 - \lambda) \beta S_t^w \left( 1 - \frac{M}{\psi} \right) \left[ E_t \tilde{u}_{Ct+1} - \tilde{u}_{Ct} + E_t \tilde{S}_{t+1}^w \right] - (1 - \lambda) \beta S_t^w \frac{M}{\psi} \left( E_t \tilde{M}_{t+1} - E_t \tilde{U}_{t+1} \right)
\end{align*} \]

\[ \begin{align*}
\text{(D.6') } & \quad \tilde{u}_{Ct} u_C = \left[ \psi^{-1} \left( \beta_t - \hat{\Psi}_t \right) + \gamma \mu \left[ C \left( 1 - \frac{h_{\lambda C}}{g_A} \right) \right] \gamma^{-1} X^\gamma g_A^{-1} \left[ \hat{\mu}_t + (1 - \gamma) \hat{X}_{t-1} + (\gamma - 1) \hat{g}_{At} \right] \\
& \quad + \gamma (\gamma - 1) \mu X^\gamma g_A^{-1} \left[ C \left( 1 - \frac{h_{\lambda C}}{g_A} \right) \right] \gamma^{-2} \left[ \tilde{C}_t C - h_{\lambda C} \frac{C}{g_A} \left( \tilde{C}_{t-1} - \tilde{g}_{At} \right) \right] \\
& \quad - \beta h \left( \psi \frac{\mu g_A}{\psi} \right) \gamma^{-1} X^\gamma C - h_{\lambda C} \frac{C}{g_A} \left( \tilde{C}_{t-1} - \tilde{g}_{At} \right) \\
& \quad - (1 - \gamma) \beta h \left( \psi \frac{\mu g_A}{\psi} \right) \gamma^{-1} X^\gamma C - h_{\lambda C} \frac{C}{g_A} \left( \tilde{C}_{t-1} - \tilde{g}_{At} \right)
\end{align*} \]

\[ \begin{align*}
\text{(D.11) } & \quad \hat{\mu}_t \mu = \left[ \psi^{-1} \left( \beta_t - \hat{\Psi}_t + L_t + \hat{h}_t + (1 + \omega) \hat{h}_t \right) \right] \\
& \quad + (1 - \gamma) \beta h \left( \frac{g_A}{g_{\lambda C}} \right) X^\gamma C - h_{\lambda C} \left( \tilde{C}_{t-1} - \tilde{g}_{At} \right) \gamma^{-1} X^\gamma C - h_{\lambda C} \left( \tilde{C}_{t-1} - \tilde{g}_{At} \right)
\end{align*} \]

\[ \begin{align*}
\text{(D.12) } & \quad \hat{\Psi}_t \Psi = \left( \tilde{C}_t C - h_{\lambda C} \frac{C}{g_A} \right) \left( \tilde{C}_{t-1} - \tilde{g}_{At} \right) - L_t h^{1 + \omega} X_t \left( \tilde{h}_t + \tilde{L}_t + (1 + \omega) \hat{h}_t + \tilde{X}_t \right)
\end{align*} \]

\[ \begin{align*}
\text{(D.13) } & \quad \hat{X}_t X = \left[ \gamma C \left( 1 - \frac{h_{\lambda C}}{g_A} \right) \gamma^{-1} \left( \frac{X}{g_A} \right)^{\gamma - 1} \left( \tilde{C}_t C - h_{\lambda C} \frac{C}{g_A} \right) \left( \tilde{C}_{t-1} - \tilde{g}_{At} \right) \right] \\
& \quad + (1 - \gamma) \left[ C \left( 1 - \frac{h_{\lambda C}}{g_A} \right) \gamma^{-1} \left( \frac{X}{g_A} \right)^{\gamma - 1} \left( \tilde{X}_{t-1} - \tilde{g}_{At} \right) \right]
\end{align*} \]

\[ \begin{align*}
\text{(D.14) } & \quad \hat{h}_t = \tilde{h}_t \\
\text{(D.15) } & \quad \Delta_{\hat{h}} \tilde{\Delta}_{ht} = \left( \hat{h}_t - \tilde{h}_t \right) - \phi_n h^2 \hat{h}_t
\end{align*} \]

Note: Other equations are unchanged relative to Table A.7.

A-36
References


