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Lumpiness, capital adjustment costs and investment dynamics

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ABSTRACT

Aggregate investment in the US economy displays a hump-shaped pattern in response to shocks, and the autocorrelation of aggregate investment growth is positive for the first few quarters, turning negative for the later quarters. This paper shows that this feature of the data is the natural outcome of a two-sector consumption/investment model designed and calibrated to reproduce plant-level evidence on capital accumulation.

1. Introduction

Aggregate investment in the US economy displays a hump-shaped pattern in response to shocks, and the autocorrelation of aggregate investment growth is positive for the first few quarters, turning negative for the later quarters. Existing one-sector models that are frictionless or that include costs to adjust the stock of capital – *capital adjustment costs* – are not consistent with this feature, while a one-sector model that includes costs to change the level of investment – *investment adjustment costs* – reproduces this behavior. Following Christiano et al. (2005), investment adjustment costs have featured prominently in the recent literature on dynamic general equilibrium models, although they seem to be at odds with plant-level evidence on capital accumulation that typically takes the form of a large and infrequent, or lumpy, episode.[1] Moreover, in contrast to the large empirical literature on capital adjustment costs, which often takes a disaggregated approach, less evidence is available for investment adjustment costs and the only attempt to estimate them at a disaggregated level is the work by Groth and Khan (2010).

In this paper I study whether the inclusion of capital adjustment costs, convex or non-convex, in an otherwise standard neoclassical two-sector model yields investment dynamics consistent with the data. Two main results emerge.

First, hump-shaped responses of aggregate investment are natural and consistent with plant-level evidence on capital accumulation. A two-sector consumption/investment model with non-convex capital adjustment costs, exclusively calibrated to reproduce cross-sectional plant-level data, generates a smooth response of aggregate investment qualitatively and statistically in line with the data. Interestingly, the series for output and hours also inherit a hump-shaped response as opposed to the spikes observed in the frictionless model. The key to the model's success is the inclusion of non-convex capital adjustment costs in the investment sector that smooths the shift in the investment supply following shocks. This modification significantly improves the short-run dynamic behavior of aggregate investment relative to the

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[1] Doms and Dunne (1998) find that between 25% and 40% of the typical plant's total investment expenditure over a 17-year sample is concentrated into a single large episode. Gourio and Kashyap (2007), using data on US manufacturing, show that cyclical fluctuations in the investment rate are driven by variations in the number of plants experiencing *spikes* (defined as an investment rate above 20%).

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frictionless setup that is instead rejected by the data. The relevance of lumpiness is in contrast to the work of Thomas (2002), Khan and Thomas (2003, 2008) and House (2008). In the context of one-sector models, they find lumpy investment to be neutral for the dynamics of aggregate investment that are indistinguishable from those of a frictionless neoclassical model. The reason is that general equilibrium effects, namely, an increase in wages and in the interest rate, dampen the increase in investment demand triggered by the shock, making the dynamics of the two models fairly identical. The innovation in my setup is that lumpiness applies to the investment sector, which can be interpreted as the investment supply. The Khan–Thomas mechanism is still at work; following an aggregate productivity shock, wages and interest rates dampen investment demand. The additional effect, absent in a one-sector model, is that now lumpiness hinders investment supply (i.e., the investment sector), preventing a quick increase in the production of the investment good, leading in equilibrium to a further increase in the relative price of investment that in turn dampens investment demand even more. General equilibrium effects, namely, the relative price of investment, are the source of the differential investment response between the frictionless and the lumpy model. Moreover, the inclusion of non-convex costs to both sectors yields virtually identical results, confirming the crucial role played by the supply of investment and the irrelevance of investment demand as in House (2008): the investment demand implied by the lumpy model is virtually equivalent to the one obtained with a frictionless neoclassical framework.

Second, a two-sector model augmented with convex capital adjustment costs in the investment sector also generates a data-consistent hump-shaped investment response. The result obtained with a reduced-form specification of the capital adjustment cost function is consistent with a realistic microfoundation of such frictions. This finding has relevant modelling implications because it provides an alternative to the specification of investment adjustment costs in order to obtain hump-shaped dynamics. It is necessary to clarify that other frictions, such as labor adjustment costs, can induce, at least qualitatively, humps in investment by preventing a rapid expansion of the investment sector in response to shocks.

The remainder of the paper is organized as follows. Sections 2 and 3 present the workings and the calibration of the models with lumpy investment and convex capital adjustment costs. Section 4 reports the simulation results following an aggregate productivity shock. Section 5 conducts an empirical test to determine the extent to which capital adjustment costs can improve the dynamic behavior of the frictionless model. Section 6 concludes the paper.

2. A two-sector model with lumpiness

This section describes the details of the model with non-convex adjustment costs. I introduce lumpiness in a two-sector neoclassical growth model. Non-convex costs of capital adjustment apply to plants in each sector. Investment decisions at the plant level are subject to a random cost drawn from a known distribution. The fixed cost induces a distribution of plants over capital levels, one for each sector. As in Thomas (2002) state-dependent investment at the plant level is modelled using a generalized (*S*,*s*) framework. The approach is related to Caballero and Engel's (1999) generalized (*S*,*s*) model in its use of stochastic adjustment costs to simultaneously yield lumpy plant-level investment and smooth aggregates. This setup allows for probabilistic adjustment thresholds that can capture the rising hazards observed in the microeconomic data. The description of the model follows Thomas (2002).

2.1. Production and investment decisions at the plant level

This subsection describes production and investment decisions in the economy. Motivated by Ramey and Shapiro (2001), who suggest significant sectoral specificity of physical capital and substantial costs of redeploying the capital, I favor a two-sector interpretation of the economy. One sector produces consumption goods and the other produces investment goods. In each sector there is a continuum of production units that are heterogeneous in their stock of capital. Plants produce using capital and labor as variable inputs. Labor can be freely adjusted in each period and is perfectly mobile across sectors, while capital adjustment is subject to a fixed labor cost. These adjustment costs, denoted by ξ, are independently and identically distributed across establishments and across time with a known cumulative distribution of $G_s(\xi)$ and finite upper support B_s. The subscript s represents the sector to which the variable refers: C for the consumption sector and K for the investment sector. The fixed cost is denominated in units of labor to ensure that plants cannot outgrow adjustment costs along the balanced growth path.

A production unit that has adjusted its level of capital j periods ago will have its capital denoted by a subscript j. All plants in both sectors share the same production technology with diminishing returns to scale with respect to the two inputs of production, capital $k_{j,t,s}$ and labor $n_{j,t,s}$. The production function is a Cobb–Douglas:

$$y_{j,t,s} = A_t k_{j,t,s}^{\gamma} n_{j,t,s}^{\nu}. \tag{1}$$

All establishments share the same productivity A_t, which is determined by the realization of a stochastic component, z_t, and a trend component, X_t. The component X_t evolves deterministically with growth rate Θ_A, and z_t follows a mean zero AR(1) process in logs with persistence ρ.

In each period, plants in each sector, after observing the realization of aggregate productivity and its individual adjustment cost, decide whether to invest. If a plant decides to adjust its stock of capital for date $t+1$, it pays its current cost draw ξw_t, in units of labor (w_t represents the real wage at date t) and invests in order to reach a target level of capital $k_{0,t+1,s}$:

$$k_{0,t+1,s} = (1-\delta)k_{j,t,s} + i_{j,t,s}; \quad s = C,K; \;\; j = 1\ldots J_s. \tag{2}$$

If an establishment stays inactive, its stock of capital next period is the depreciated current level of capital:

$$k_{j+1,t+1,s} = (1-\delta)k_{j,t,s}; \quad s = C,K; \;\; j = 1\ldots J_s. \tag{3}$$

Each plant's current flow of profit is determined by its total revenues less wage payments, investment and adjustment costs. Given diminishing returns, plants make profits that are rebated to households in lump-sum fashion.

2.2. Aggregation at the sectoral level

In each sector, all plants have the same technology and face the same distribution of adjustment costs. As a result, they share the same expected stream of future marginal revenues for any given choice of future capital. Thus, investors choose a common target capital $k_{0,t+1,s}$. All plants adjusting at a given time are identical immediately after investing. The cross-sectional distribution of establishments over capital levels is therefore summarized by the distribution of plants across *time-since-adjustment*, or, loosely, vintages, where each member of a group shares the same time since the last capital adjustment and it is associated with the same capital stock. There are two distributions of plants, one for each sector, over levels of capital. Each group, or *vintage j*, contains a marginal plant at whose cost draw it is just worthwhile to invest. All plants of the same group that are drawing costs at or below this group-specific threshold also invest, implying that the investing fraction for each group, $\alpha_{j,t,s}$, is retrievable from the adjustment cumulative density function (*cdf*).

At each date, the sectoral distribution of the economy's establishments across sectors is summarized by two vectors: $\mathbf{k}_{t,s}$ and $\boldsymbol{\vartheta}_{t,s}$. The vector $\mathbf{k}_{t,s} = \{k_{j,t,s}\}$ represents the vector of capital levels across time-since-adjustment groups. The fraction of plants associated with each level of capital is given by the predetermined vector $\boldsymbol{\vartheta}_{t,s} = \{\vartheta_{j,t,s}\}$. Each $\vartheta_{j,t,s}$ describes the number of firms owning vintage j capital stock. The evolution of the sectoral distribution is determined as follows. The support at time $t+1$ is determined through (3) and adjusting plants' common choice of $k_{0,t+1,s}$. Let $\boldsymbol{\alpha}_{t,s} = \{\alpha_{j,t,s}\}$ denote the fraction of adjustment rates. The evolution of the distribution of plants $\boldsymbol{\vartheta}_{t+1,s}$ is determined by the following equations:

$$\vartheta_{0,t+1,s} = \sum_{j=0}^{Js} \alpha_{j,t,s}\vartheta_{j,t,s}; \quad s = C,K, \tag{4}$$

$$\vartheta_{j+1,t+1,s} = (1-\alpha_{j,t,s})\vartheta_{j,t,s}; \quad s = C,K; \;\; j = 1,2\ldots J_s. \tag{5}$$

The group of plants that have adjusted their stock of capital in the current period is the weighted sum of adjusters in each group. If a plant decides not to adjust at date t, it becomes vintage $t+1$ in the subsequent period. The total stock of capital in the economy is the sum of the stock of capital in each sector.

2.3. Households

A representative household owns the portfolio of plants in both sectors and supplies labor. The household values consumption and leisure according to a momentary utility function $u(C_t, 1-N_t)$, where N_t is the fraction of time devoted to market activity, and discounts future utility by the factor β. Consumption is financed by labor income and profits received from the plants.

2.4. Aggregate constraints

The economy is subject to a set of aggregate constraints. Household consumption cannot exceed the total production of the consumption good:

$$C_t \le \sum_{j=0}^{J_C} \vartheta_{j,t,C} y_{j,t,C}. \tag{6}$$

Aggregate investment cannot exceed the production of the investment sector:

$$I_{t,C} + I_{t,K} \le \sum_{j=0}^{J_K} \vartheta_{j,t,K} y_{j,t,K}, \tag{7}$$

where $I_{t,C}$ ($I_{t,K}$) is the total sectoral investment in the consumption (investment) sector:

$$I_{t,C} = \sum_{j=0}^{J_C} \vartheta_{j,t,C}\alpha_{j,t,C} i_{j,t,C} \quad \text{and} \quad I_{t,K} = \sum_{j=0}^{J_K} \vartheta_{j,t,K}\alpha_{j,t,K} i_{j,t,K}. \tag{8}$$

Total hours worked by the household must satisfy the weighted sum of employment in production and adjustment activities in each sector:

$$\sum_{j=0}^{J_C} \vartheta_{j,t,C} n_{j,t,C} + \sum_{j=0}^{J_C} \vartheta_{j,t,C} \Xi(\alpha_{j,t,C}) + \sum_{j=0}^{J_K} \vartheta_{j,t,K} n_{j,t,K} + \sum_{j=0}^{J_K} \vartheta_{j,t,K} \Xi(\alpha_{j,t,K}) \le N_t. \tag{9}$$

Below is the average adjustment cost for each group in each sector.

$$\Xi(\alpha_{j,t+1,s}) = \int_0^{G_s^{-1}(\alpha_{j,t,s})} x \, dG_s(x). \tag{10}$$

3. Planner's problem

Competitive equilibrium allocations are determined through the solution of a planning problem. Specifically, the planner maximizes $E_0 \sum_{t=0}^{\infty} \beta^t [u(C_t, 1-N_t)]$ subject to the production function (1), the laws of motion for capital in investing (2) and non-investing plants (3), the laws of motion for the distribution of plants (4) and (5), and the constraints for sectoral output (6) and (7), together with the aggregate labor constraint (9).

3.1. Optimal consumption and labor allocation

The solution satisfies a series of efficiency conditions for consumption and labor decision:

$$\lambda_t = D_1 u(C_t, 1-N_t). \tag{11}$$

The first-order condition for consumption (C_t) in Eq. (11) equalizes the multiplier on the constraint of Eq. (6) with the marginal utility of consumption, where D represents the first derivative of the utility function and the subscript denotes the argument with respect to which derivative is taken:

$$w_t = \frac{D_2 u(C_t, 1-N_t)}{D_1 u(C_t, 1-N_t)}. \tag{12}$$

Total labor hours equate the marginal rate of substitution between leisure and consumption to w_t, the multiplier on the time constraint (9), which represents the real wage. Plant-level employment in production satisfies the static condition for labor, for $j = 1 \ldots J_s$:

$$w_t = \left(\frac{\nu y_{j,t,C}}{n_{j,t,C}} \right), \tag{13}$$

$$w_t = \left(\frac{\nu p_t y_{j,t,K}}{n_{j,t,K}} \right). \tag{14}$$

Each plant in both sectors hires labor until the marginal product of labor is equal to the real wage, where p_t is the ratio between the multiplier attached to the constraints (6) and (7) and represents the relative price of an additional unit of capital.

3.2. Optimal adjustment thresholds

In this subsection the efficiency conditions define optimal adjustment fractions. The finite upper support for the cost *cdf*, combined with a constant rate of depreciation, makes investment increasingly valuable across vintages. As in Thomas (2002), this result greatly simplifies the state space of the model, and the economic history is redundant beyond a finite number of lags. Once the capital stock has depreciated enough, the value of investing offsets the highest possible fixed cost. In equilibrium, there exists in each sector an endogenously chosen vintage J_s at which full adjustment occurs: $\alpha_{J_s} = 1$. In each sector, for $j_s < J_s$, the optimal fractions of adjusting are interior solutions equating the anticipated value of adjusting one additional plant from group j to the additional cost entailed, $w_t G^{-1}(\alpha_{j,t,s})$ in units of labor, and investment required. For each $j_s = 0 \ldots J_s - 1$:

$$v_{0,t,s} - v_{j+1,t,s} = p_t i_{j,t,s} + w_t G^{-1}(\alpha_{j,t,s}), \tag{15}$$

$$v_{j,t,C} = E_t\{\beta_{t,t+1}[y_{j,t+1,C} - w_{t+1}(n_{j,t+1,C} + \Xi(\alpha_{j,t+1,C})) - \alpha_{j,t+1,C}(p_{t+1} i_{j,t+1,C} - v_{0,t+1,C} + v_{j+1,t+1,C}) + v_{j+1,t+1,C}]\}, \tag{16}$$

$$v_{j,t,K} = E_t\{\beta_{t,t+1}[p_{t+1} y_{j,t+1,K} - w_{t+1}(n_{j,t+1,K} + \Xi(\alpha_{j,t+1,K})) - \alpha_{j,t+1,K}(p_{t+1} i_{j,t+1,K} - v_{0,t+1,K} + v_{j+1,t+1,K}) + v_{j+1,t+1,K}]\}, \tag{17}$$

where $\beta_{t,t+1}$ is equal to $\beta(\lambda_{t+1})/\lambda_t$ while $v_{0,t,C}$ ($v_{0,t,K}$) is the multiplier on the constraints for the target capital in the consumption (investment) sector (2) and it represents the marginal value of having one more plant with the target level of capital in the next period. $v_{j,t,C}(v_{j,t,K})$ is the multiplier on the constraints (3) in the consumption (investment) sector and represents the marginal value of having one more plant in vintage j in the next period. The values of the multipliers $\upsilon_{j,t,C}$ and $\upsilon_{j,t,K}$ are defined in Eqs. (16) and (17). Plants in the consumption sector sell their output (the consumption good) at a price normalized to 1, and plants in the investment sector sell their output at p, the price of investment relative to the consumption good. There will be a marginal firm for which Eq. (15) holds with equality that identifies $\alpha_{j,C}$ and $\alpha_{j,K}$, the fraction of plants for given vintage j that decides to invest and can be interpreted as hazard rates. $\alpha_{j,C}$ and $\alpha_{j,K}$ are increasing in j, the number of periods of inaction of a plant. The higher the gap between the target and the actual capital, the higher the probability that a plant will invest.

3.3. Optimal sectoral capital allocation

Sectoral capital allocation is subject to the following efficiency conditions:

$$\mu_{0,t,C} = p_t, \tag{18}$$

$$\mu_{0,t,K} = p_t. \tag{19}$$

For each $j = 1 \ldots J_s - 1$:

$$\mu_{j,t,C} = E_t\left\{\beta_{t,t+1}\left[\frac{\gamma y_{j,t+1,C}}{k_{j,t+1,C}} + (1-\delta)((1-\alpha_{j,t+1,C})\mu_{j+1,t+1,C} + \alpha_{j,t+1,C}p_{t+1})\right]\right\} \tag{20}$$

and

$$\mu_{j,t,K} = E_t\left\{\beta_{t,t+1}\left[p_{t+1}\frac{\gamma y_{j,t+1,K}}{k_{j,t+1,K}} + (1-\delta)((1-\alpha_{j,t+1,K})\mu_{j+1,t+1,K} + \alpha_{j,t+1,K}p_{t+1})\right]\right\}. \tag{21}$$

Eqs. (18) and (19) govern the installation of new capital in each sector. The investment good is allocated to each sector until $\mu_{0,t,C}$, the expected marginal value of installing an additional unit of capital in the consumption sector, defined by Eq. (20), and $\mu_{0,t,K}$, the expected marginal value of installing one additional unit in the investment sector, defined by Eq. (21), are equal to p_t. $\mu_{j,t,s}$ denotes the marginal value of an additional unit of capital in vintage j. The lumpy model nests as a specific case the two-sector frictionless model. When the upper supports of the distribution of the idiosyncratic shock in each sector, B_C and B_K, are set to zero, the planner's problem boils down to a two-sector neoclassical model with two representative firms: one producing the consumption good and one producing the investment good.[2]

3.4. Model solution

This subsection describes the algorithm used to solve the model. The procedure builds on Thomas (2002) and Gourio and Kashyap (2007). The computation of the steady state requires a numerical procedure because J_C and J_K, maximum time-since-adjustment in the consumption and in the investment sector, are endogenously determined. The first step in the solution algorithm is guessing the target capital in both sectors $k_{0,C}$ and $k_{0,K}$, the relative price of investment p, the fraction of plants investing for each level of capital $\boldsymbol{\alpha}_{j,C}$ and $\boldsymbol{\alpha}_{j,K}$ and the real wage w. After plugging the values into the equations, I use (7), (13), (16)–(19) to verify and update the guessed values. This procedure continues until α_{J_C} and α_{J_K} are equal to 1 (i.e., plants with level of capital k_{J_C} and k_{J_K} adjust with probability 1).

3.5. Calibration

The parameterization is reported in Table 1. The calibration strategy tightly follows Thomas (2002), the only exception being that the period length is one quarter as opposed to 1 year.[3] The momentary utility function is $u(C_t, 1-N_t) = \log(C_t) - \zeta N_t$.[4] The parameter of the disutility of labor (ζ) is chosen so that labor supply is 0.2 in the steady state. The discount factor in the utility function is set to obtain an annual interest rate of 6.5%, given long-run output growth of 1.6% per year; see King and Rebelo (1999). The rate of capital depreciation (δ) is set to 0.015, the quarterly counterpart of the annual rate assumed in Thomas. There are decreasing returns to scale in production ($\gamma + \nu = 0.905$) as in Thomas (2002); labor's share of output is 0.58 (King and Rebelo, 1999). The standard deviation of the aggregate technology shock is chosen so that the frictionless model reproduces the volatility of output in the data; this yields $\sigma_\epsilon = 0.0089$. The persistence of the aggregate productivity shock is set to 0.979 as in King and Rebelo (1999).

The remaining parameters involve the parameterization of the adjustment cost function for the state-dependent lumpy model. The cumulative distribution for adjustment costs is set to imply a uniformly distributed cost between 0 and B_C (B_K) for the consumption (investment) sector. The upper support of the distributions (B_C and B_K) is set to agree with evidence on investment spikes reported by Doms and Dunne (1998). In the average year, 8% of plants raise their capital stocks by

[2] See, for instance, Dow and Olson (1992) and Boldrin et al. (2001). In addition you need to assume that sectoral investment must be non-negative.

[3] This choice is motivated by the focus on business cycle analysis and to avoid loss of information that comes from temporal aggregation of quarterly data into yearly data (see Cogley and Nason, 1995 for a discussion).

[4] Labor is indivisible, as in Hansen (1985).

Table 1
Calibration.

Parameter	Value
Depreciation rate (δ)	0.015
Persistence of the TFP Shock (ρ)	0.979
Returns to scale ($\psi+v$)	0.905
B_c maximum fixed cost consumption sector	0.007
B_k maximum fixed cost investment sector	0.0018
Discount factor (β)	0.989
Intertemporal elasticity of substitution	1
Frisch elasticity of labor supply	Infinite
Standard deviation of the productivity shock	0.0089

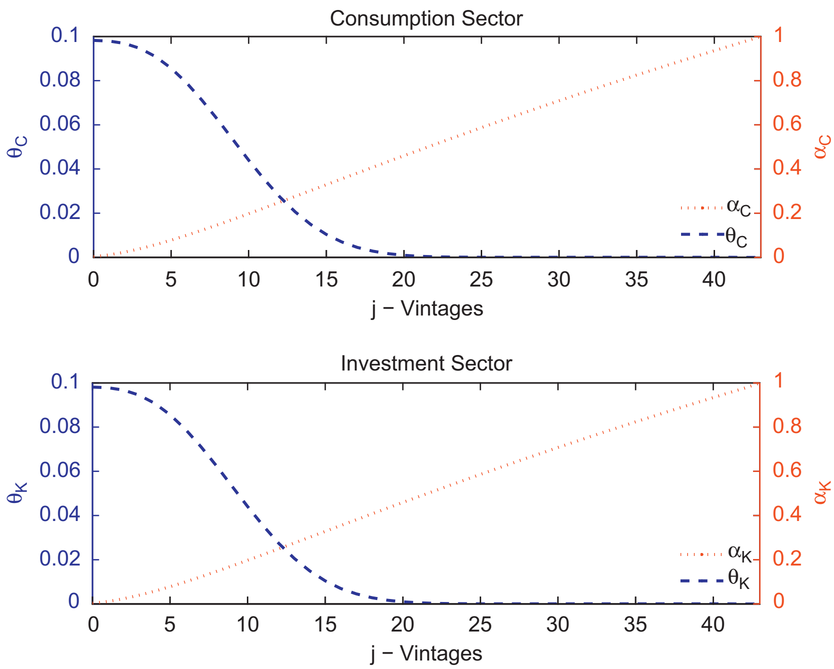

Fig. 1. The panels show the steady-state capital distribution in each sector ($\vartheta_{j,C}$ and $\vartheta_{j,K}$) together with the hazard rates ($\alpha_{j,C}$ and $\alpha_{j,K}$).

30% or more (lumpy investors). Setting $B_C = 0.0073$ and $B_K = 0.0018$ matches the fraction of lumpy investors, while their investment activities in the model account for 31% of aggregate investment (25% in the data).[5] The implied adjustment cost is equal to 0.1% of aggregate investment, a value comparable with Thomas (2002).[6] Panels in Fig. 1 report the steady-state distributions of firms (θ_C and θ_K) and the hazard rates (α_C and α_K).

The maximum time-since-adjustment J_C and J_K is equal to 43 in both sectors; in other words the maximum time-since-adjustment for a plant is about 10 years.

4. A two-sector model with convex adjustment costs

This section describes the workings of a two-sector model with convex costs to adjust capital in the investment sector. In contrast with the model with lumpiness, each sector features a representative firm as opposed to heterogeneous units. As in the lumpy model, both sectors share technology and the same Cobb–Douglas production function. I follow Basu et al. (2001) and Ireland and Schuh (2008) in assuming that adjustment costs are denominated in units of sectoral output, and this modifies the aggregate resource constraints as follows:

$$C_t \leq Y_{t,C} \tag{22}$$

[5] Given the quarterly calibration of the model, the target for lumpy investors is 2% at quarterly frequency: plants that are lumpy investors in period t (they experience an investment rate above 30%) along the balanced growth path will again be lumpy investors beyond period $t+4$.

[6] In Thomas, the ratio of total adjustment cost to total investment is equal to 0.2%. Cooper and Haltiwanger (2006) estimate the ratio to be 7.5%; see Gourio and Kashyap (2007) for a discussion.

and

$$I_t \leq Y_{t,K}\left[1-\frac{\phi_K}{2}\left(\frac{I_{t,K}}{K_{t,K}}-\kappa_K\right)^2\right], \tag{23}$$

where the non-negative parameter ϕ_K governs the magnitude of the adjustment costs, and the parameter κ_K ensures that steady-state adjustment costs equal zero. $I_{t,C}$ ($I_{t,K}$) is the total sectoral investment in the consumption (investment) sector. The sum of sectoral investment ($I_{t,C}+I_{t,K}$) cannot exceed total output of the investment sector I_t. Total hours worked by the household must satisfy the sum of employment in production in each sector:

$$N_{t,C}+N_{t,K} \leq N_t. \tag{24}$$

The law of motion for sectoral capital is consistent with that in the previous section, in each sector:

$$K_{t+1,s} = (1-\delta)K_{t,s}+I_{t,s}, \quad s=C,K. \tag{25}$$

4.1. Equilibrium allocations

Equilibrium allocations are determined through the solution of a planning problem. Specifically, the planner maximizes $E_0\sum_{t=0}^{\infty}\beta^t[u(C_t,1-N_t)]$ subject to (1), (22)–(25). The solution satisfies a series of efficiency conditions:

$$\lambda_t = D_1 u(C_t,1-N_t). \tag{26}$$

The first-order condition for consumption (C_t) in Eq. (26) equalizes λ_t, the multiplier on the constraint of Eq. (22), with the marginal utility of consumption. The marginal rate of substitution between leisure and consumption is equal to the multiplier on the time constraint (24), which represents the real wage. Sectoral capital allocation is subject to the following efficiency conditions, where p_t is the ratio between the multiplier attached to constraints (22) and (23) and represents the relative price of an additional unit of capital:

$$\mu_{t,C} = p_t, \tag{27}$$

$$\mu_{t,K} = p_t\left[1+\phi_{KK}\left(\frac{I_{t,K}}{K_{t,K}}-\kappa_K\right)\frac{I_t}{K_{t,K}}\right], \tag{28}$$

$$\mu_{t,C} = E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\left[\gamma\frac{C_{t+1,C}}{K_{t+1,C}}+(1-\delta)\mu_{t+1,C}\right]\right\}, \tag{29}$$

$$\mu_{t,K} = E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\left[\gamma\frac{p_{t+1}I_{t+1}}{K_{t+1,K}}+(1-\delta)\mu_{t+1,K}+\phi_{KK}\left(\frac{I_{t+1,K}}{K_{t+1,K}}-\kappa_K\right)\left(\frac{I_{t+1,K}}{K_{t+1,K}}\right)\frac{I_{t+1}}{K_{t+1,K}}\right]\right\}. \tag{30}$$

Eqs. (27) and (28) govern the installation of new capital in each sector. The investment good is allocated to each sector until $\mu_{t,C}$, the expected marginal value of installing an additional unit of capital in the consumption sector, defined by Eq. (29), and $\mu_{t,K}$, the expected marginal value of installing one additional unit in the investment sector, defined by Eq. (30), are equal to p_t. Eq. (28) drives a Q-theoretic wedge between the shadow price, p_t, of newly produced investment goods and $\mu_{t,K}$, the shadow price of installed capital in the investment sector. When ϕ_K is set to zero, the planner's problem boils down to a two-sector neoclassical model with two representative firms: one producing the consumption good and one producing the investment good.

4.2. Calibration

The calibration of the convex model follows the one of the frictionless model reported in Table 1. The additional parameter that needs to be specified is ϕ_K, which governs the adjustment cost function for capital. ϕ_K is set to imply an elasticity of the sectoral investment rate to Tobin's Q equal to 11, a value consistent with Hall (2004).[7]

5. Simulation results

This section reports a numerical analysis of the models presented in Sections 2 and 3. The equilibrium conditions are linearized around the deterministic (growth deflated) steady state, and linear decision rules are then computed using the method in Klein (2000). Then, I analyze the impulse response functions after a 1% increase in aggregate productivity for the lumpy model and the model with convex capital adjustment costs relative to a frictionless two-sector reference.[8]

[7] This value is on the high end of the ones used in the literature. Among others, Jermann (1998) estimates this elasticity to be around 4.34, while Shapiro (1986) finds values between 8 and 9.

[8] Greenwood et al. (1988, 2000) and Fisher (2006) have highlighted the importance of investment-specific productivity shocks as the driving force of business cycle fluctuations. The results discussed below are not affected when the source of fluctuations in the model is an investment-specific shock.

5.1. Dynamics – lumpy model

This subsection analyzes the dynamics implied by a frictionless setup with those of a model featuring lumpy investment. Fig. 2 reports the impulse response functions. The lumpy model is able to reconcile hump-shaped aggregate investment responses with plant-level evidence on capital accumulation. The key to the model's success is the inclusion of lumpiness in the investment sector, which represents the supply of the investment good. This modification improves the ability of a frictionless two-sector model to generate a hump-shaped investment response. Such improvement is induced by non-convex adjustment costs that smooth capital accumulation in the investment sector and prevent a rapid expansion of production of the investment good. This dynamic behavior generates a hump in investment, and a similar one in hours, output and consumption (although reversed) in contrast to the frictionless model. By the same token, clearing in the investment good market requires a larger increase in the relative price of investment with respect to the frictionless framework. The shift of the investment supply in the frictionless setup is instead sharp, leading one period after the shock to spikes in aggregate investment, hours, output and consumption (although reversed). The similar response displayed on impact by the relative price of investment is linked to the close behavior of investment supply in the two models following the shock. Labor and aggregate productivity (capital is predetermined) boost sectoral output (i.e., increase the investment supply) by virtually the same amount. As discussed below, the inclusion of lumpiness in the consumption sector is irrelevant because both the frictionless and the lumpy model imply a similar investment demand. Therefore, given the equivalence of investment demand and the close behavior of investment supply, clearing in the investment good market implies in the two setups a similar response of the relative price of investment. Despite the fact that aggregate investment and the relative price of investment are, on impact, quantitatively close, sectoral allocation of the investment good in the lumpy model is different from the frictionless framework. All in all, the lumpy model designed to reproduce cross-sectional evidence on micro-data reconciles the gradual response of aggregate investment with the lumpy nature of capital accumulation observed at the plant level.

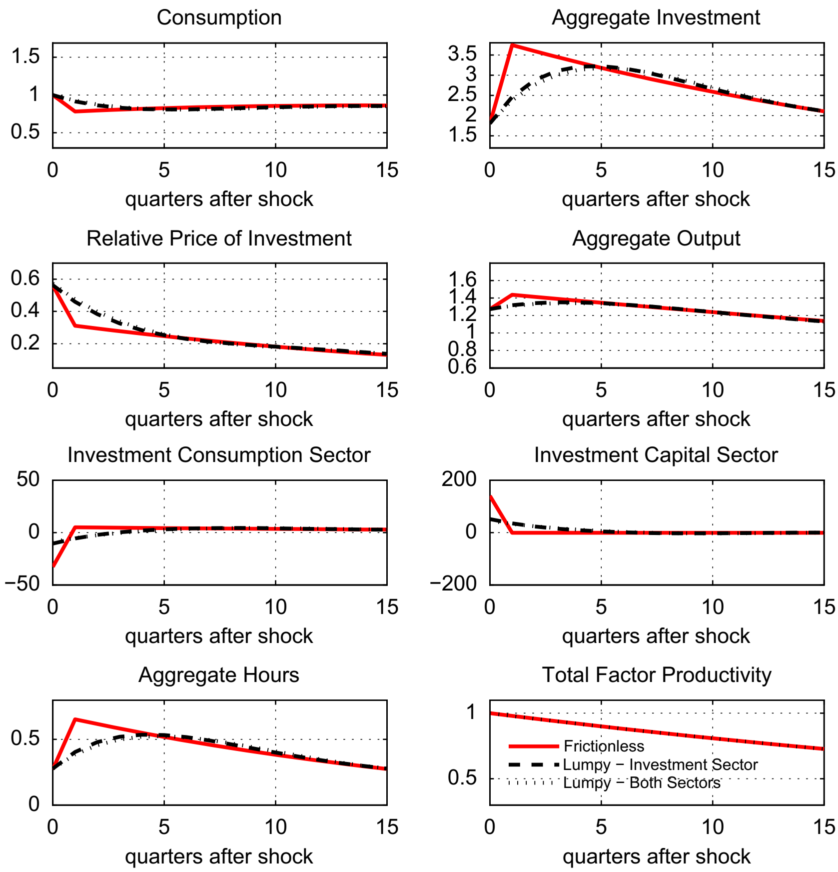

Fig. 2. Impulse responses to a 1% increase in aggregate productivity implied by the two-sector frictionless and the two-sector lumpy model. Responses are reported in deviations from steady state.

5.2. Relation with previous literature

The non-neutrality of lumpiness is in contrast with the work of Khan and Thomas and House. Khan and Thomas show that, in one-sector models, general equilibrium effects, namely, an increase in wages and the interest rate, dampen the increase in investment demand triggered by the shock and yield equivalent dynamics for both the lumpy and the frictionless model. The Khan–Thomas mechanism is still at work in my setup but there is an additional effect: lumpiness hinders the shift in investment supply, leading to a further increase in the relative price that dampens investment demand even more relative to the frictionless case. Even in a multisector economy, general equilibrium effects dampen the response of aggregate investment compared to partial equilibrium frameworks; however, the conclusion that investment dynamics in the lumpy and in the frictionless model are indistinguishable does not follow. Lumpiness acts as a smoothing force for aggregate investment and delivers hump-shaped investment responses *because* of general equilibrium effects through the relative price of investment. House (2008) parameterizes the equivalent in my setup of a frictionless investment sector (frictionless supply) and a lumpy consumption sector. He finds that the investment demand implied by the lumpy model is equivalent to that of a frictionless neoclassical model. Introducing lumpiness in the consumption sector, with a frictionless or a lumpy investment sector (shown in Fig. 2), does not quantitatively alter the responses, confirming the role of supply in driving investment dynamics. In addition, while in House the relative price of investment barely moves, in my framework, variations are much larger even in the absence of lumpiness, suggesting that the two-sector assumption (through sectoral specificity of capital) reduces the possibility for plants to perfectly retime their investment plans.

It is important to clarify that the mechanism at work is different from that in Bachmann et al. (2010). They focus on explaining the time-varying impulse response function of aggregate investment. Their framework features a multisector environment where plants follow (*S*,*s*) capital adjustment rules and are subject to idiosyncratic and sectoral shocks, but the relative price is assumed to be exogenous. They find that a proper accounting of partial and general equilibrium smoothing (respectively, microeconomic lumpiness and consumption smoothing), at the core of their calibration strategy, generates *history-dependent* responses of aggregate investment through variations in the cross-sectional distribution of capital holdings across plants. In the present paper the cross-sectional distribution is still a time-varying object; however, hump-shaped responses are not the product of a particular calibration. For instance, the hump-shaped result survives with a calibration according to Thomas (2002), where the distribution does not move much or with a parameterization according to Gourio–Kashyap (2007), who emphasize variations in the cross-sectional distribution along the extensive margin.

5.3. Dynamics – convex model

Fig. 3 compares the dynamics obtained in the frictionless setup with those of a model with a frictional investment sector augmented with convex capital adjustment costs. This setup represents an approximation of the lumpy model, given the irrelevance of including lumpiness in the consumption sector that is therefore assumed to be frictionless.

Hump-shaped responses of investment can be obtained even with a convex model because adjustment costs reduce the reallocation of factors across sectors and prevent a rapid expansion of the investment sector. In this context, convex capital adjustment costs, assumed to be quadratic, are a viable alternative to the increasingly popular specification of investment adjustment costs. The latter specification delivers hump-shaped predictions for aggregate investment, although its microfounded appeal is questionable. Moreover, to date the only attempt to estimate investment adjustment costs using disaggregated data is the work by Groth and Khan (2010). They find small costs associated with changing the flow of investment and suggest care in interpreting results that hinge on a structural interpretation of large investment adjustment costs.

The lumpy model is useful to provide a guide for the calibration of the convex adjustment cost function. Interestingly, the value of ϕ_{KK} that following a single/impulse shock generates virtually identical dynamics with the lumpy model implies an elasticity of the investment rate relative to Tobin's Q of about 34. This number is of the same order of magnitude as the value estimated by Cooper and Haltiwanger (2006).

6. Taking the models to the data

This section presents an empirical test to determine the extent to which the inclusion of capital adjustment costs can improve the performance of a two-sector model to account for the investment behavior observed in the data relative to the frictionless two-sector benchmark. The chosen empirical target is the hump-shaped response that aggregate investment displays in the data. As in Cogley and Nason (1995), I use as a metric the autocorrelation function (*ACF*) for investment growth.[9] In the data the hump-shape corresponds to a positive autocorrelation for the first few quarters, which then turns negative for the later quarters. To test the match between the sample and theoretical autocorrelation functions, generalized *Q-statistics* are computed.[10] The sample autocorrelations are obtained using data from 1953:1 to 2009:4.

[9] Christiano et al. (2003) and Dedola and Neri (2007) find that aggregate investment follows a hump-shape after an aggregate productivity shock. See the discussion in Boldrin et al. (2001) about the autocorrelation function of output growth in two-sector models.

[10] Q statistics are defined as $Q_{acf} = (\hat{\mathbf{c}}-\mathbf{c})'\hat{\mathbf{V}}_c^{-1}(\hat{\mathbf{c}}-\mathbf{c})$. The vectors $\hat{\mathbf{c}}$ and $\mathbf{c}$ are the sample and the model-generated autocorrelation function, respectively. The latter is estimated by averaging across the simulated samples. The covariance matrix $\hat{\mathbf{V}}_c$ is estimated by taking the ensemble outer

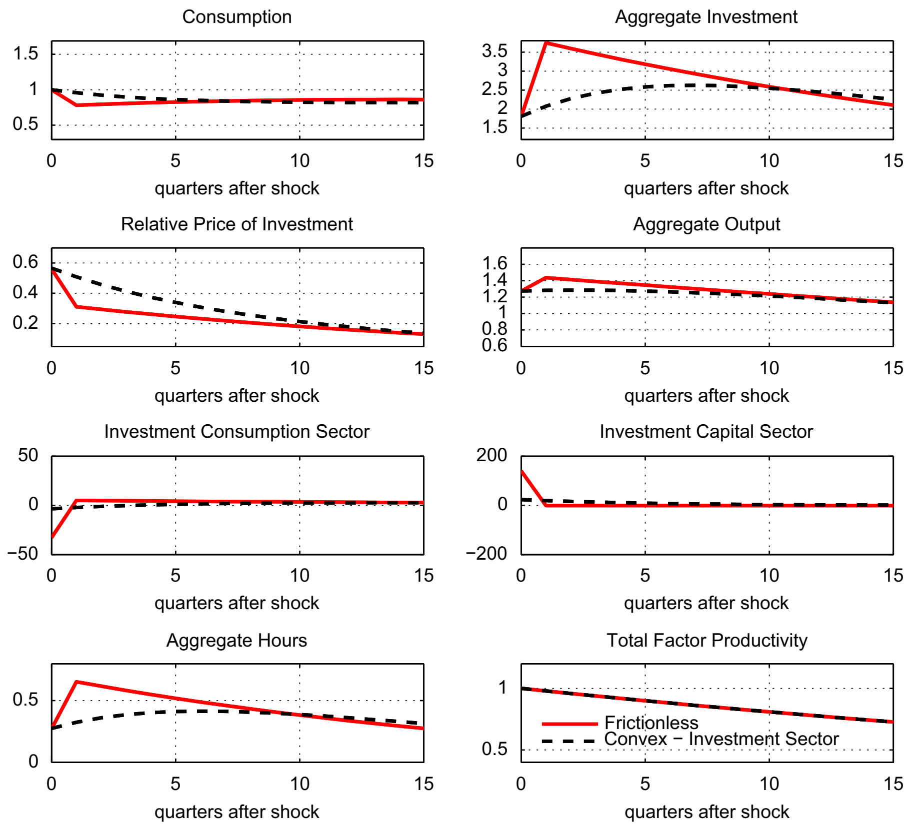

Fig. 3. Impulse responses to a 1% increase in aggregate productivity implied by the two-sector frictionless and the two-sector model with a frictional investment sector with convex adjustment costs. Responses are reported in deviations from steady state.

The theoretical autocorrelations are obtained simulating the model.[11] Table 2 reports the results. Both the lumpy and the convex model are able to reproduce a data-consistent *ACF* of investment growth, while the frictionless model is rejected at the 1% level of confidence. The lumpy model represents an upgrade relative to the basic framework in accounting for the other series analyzed, although only for hours is the test close to being significant at the 5% level. The inclusion of non-convex capital adjustment costs generates a positive autocorrelation (although close to zero) for output growth for the first three lags and reduces the negative autocorrelation of consumption. The convex model with the baseline parameterization cannot generate a positive autocorrelation of output and hours growth. Smaller adjustment costs (i.e., an even higher elasticity of the investment rate to Tobin's Q) fix this problem and yield results similar to those of the lumpy model.

Table 2 also reports the results for an alternative parameterization of the lumpy model. Since there is no agreement on how to calibrate the upper support of the distribution from which fixed costs are drawn, theoretical autocorrelations are recalculated increasing B_C and B_K by 50% relative to the baseline case.[12] Results are also not affected when the sample considers only the period of the Great Moderation (from 1984:1 on).

I now consider the implications of the models for the set of standard business cycle statistics reported in Table 3. The source of volatility is an aggregate productivity shock. The frictionless and the lumpy setup perform fairly similarly, with minor differences in the standard deviations of simulated series, while overall the convex model implies lower volatilities for all the series except consumption. It is worth noting that two-sector models fall short of the investment volatility observed in the data due to the assumption of sectoral capital specificity. Relative to a one-sector model, capital installed in the investment sector cannot later be used to produce consumption goods. As a result, the response of aggregate investment in the aftermath of a shock is dampened. In addition, lumpiness smooths the response of investment, which results in a slightly lower volatility with respect to the frictionless case, while the opposite is true for the consumption series.

(*footnote continued*)
product of the autocorrelation function for simulated data. Generalized Q statistics are approximately χ^2 with degrees of freedom equal to the number of elements in **c**. I follow Cogley and Nason (1995) in choosing eight lags and the results do not hinge on this choice.

[11] Each model's ACF was calculated by averaging this correlation over 1000 simulations (each 184-periods long as the sample in the data). Correlation and second moment statistics were calculated on raw-model data.

[12] See Bachmann et al. (2010) for a thoughtful discussion.

Table 2
Taking the models to the data.

Variable	Data	Two-sector frictionless	Two-sector lumpy baseline	Two-sector lumpy high B_C, B_K	Two-sector convex baseline
Corr(ΔI_t,ΔI_{t-1})	0.21	0.41	0.38	0.35	0.02
Corr(ΔI_t,ΔI_{t-2})	0.10	−0.16	0.26	0.25	−0.02
Corr(ΔI_t,ΔI_{t-3})	0.03	−0.14	0.17	0.17	−0.03
Q_{acf}		2952.66	12.28	11.06	11.25
p-value		0.00	0.16	0.20	0.19
Corr(ΔY_t,ΔY_{t-1})	0.36	0.06	0.03	0.03	−0.07
Corr(ΔY_t,ΔY_{t-2})	0.21	−0.10	0.02	0.02	−0.07
Corr(ΔY_t,ΔY_{t-3})	0.04	−0.08	0.02	0.02	−0.06
Q_{acf}		74.40	29.48	30.86	118.86
p-value		0.00	0.00	0.00	0.00
Corr(ΔC_t,ΔC_{t-1})	0.34	−0.27	−0.07	−0.06	−0.09
Corr(ΔC_t,ΔC_{t-2})	0.26	−0.04	−0.05	−0.05	−0.08
Corr(ΔC_t,ΔC_{t-3})	0.23	−0.03	−0.02	−0.02	−0.07
Q_{acf}		763.08	97.91	93.58	357.28
p-value		0.00	0.00	0.00	0.00
Corr(ΔH_t,ΔH_{t-1})	0.65	0.38	0.48	0.44	0.05
Corr(ΔH,ΔH_{t-2})	0.38	−0.15	0.32	0.31	0.01
Corr(ΔH_t,ΔH_{t-3})	0.16	−0.14	0.20	0.21	−0.02
Q_{acf}		111.95	15.76	18.95	96.71
p-value		0.00	0.05	0.02	0.00

Notes: This table reports the autocorrelation function for investment growth up to the third lag. Series are raw-model data. The data-values are calculated from the sample 1953:1 to 2009:4. The autocorrelation function in the Q-statistic includes eight lags as in Cogley and Nason (1995).

Table 3
Business cycle volatilities.

Variable	Data	Two-sector frictionless	Two-sector lumpy baseline	Two-sector convex baseline
σ_Y	1.59	1.59	1.54	1.46
σ_C/σ_Y	0.79	0.61	0.67	0.74
σ_I/σ_Y	4.57	2.51	2.18	1.70
σ_H/σ_Y	1.17	0.44	0.38	0.28
Corr(Y_t,Y_{t-1})	0.85	0.76	0.74	0.71
Corr(C_t,C_{t-1})	0.87	0.57	0.63	0.68
Corr(I_t,I_{t-1})	0.80	0.84	0.88	0.80
Corr(N_t,N_{t-1})	0.90	0.83	0.89	0.83
Corr(Y_t,C_t)	0.88	0.97	0.98	0.99
Corr(Y_t,I_t)	0.91	0.95	0.94	0.97
Corr(Y_t,H_t)	0.88	0.94	0.92	0.96

Notes: Data values are calculated from the sample 1953:1-2009:4 Series are logged and HP-filtered with a smoothing parameter equal to 1600. σ denotes standard deviation of a variable.
The source of fluctuations in the models is an aggregate productivity shock.

The inclusion of investment-specific shocks, an important source of business cycle fluctuations,[13] brings the volatility of investment in line with the data with little effect on the other series.[14] The response of hours is much smaller than in the data, as is usual in real business cycle models. The inclusion of capital adjustment costs delivers autocorrelations of investment and hours roughly in line with the data, while the ones for output and consumption are smaller than in the data, a result consistent with Table 2. The correlation between output and the other time series is very similar across the specifications proposed and above what is observed in the data. All in all, the inclusion of capital adjustment costs in a two-sector model significantly improves the short-run dynamic behavior of aggregate investment relative to the frictionless setup, while its performance in terms of volatilities is comparable.

[13] See Justiniano et al. (2010) for a discussion on the quantitative importance of investment-specific shocks for business cycle fluctuations.

[14] The response of aggregate investment is invariant to aggregate or investment-specific productivity shocks. Instead consumption and thus output respond less with an investment-specific shock because the productivity boost occurs only in the investment sector. Therefore the inclusion of investment-specific shocks increases the volatility of investment with little effect on the volatility of consumption and output.

7. Final remarks

Hump-shaped investment responses are natural and consistent with the lumpiness of capital accumulation observed at the plant level. The result obtained with a convex specification of capital adjustment costs is consistent with a microfoundation of investment frictions. A two-sector model with convex capital adjustment costs thus provides an alternative to the inclusion of investment adjustment costs in a one-sector setup to generate a hump in investment.

In contrast to the previous literature, lumpiness in a two-sector model is not neutral. The key to the model's success is investment supply rather than investment demand: general equilibrium effects, namely, the relative price of investment, contribute to rather than dampen the differences in the propagation of shocks relative to a frictionless framework.

A few caveats are in order, though. From Khan and Thomas (2008) the current literature on lumpy investment employs numerical models that feature idiosyncratic productivity shocks at the plant level and solution methods that allow for non-linear policy functions. These two elements are absent in my framework, but I conjecture that the main mechanism would be preserved even in more sophisticated setups. This clearly warrants future research.

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